

**London School of Hygiene & Tropical Medicine  
BASIC MATHS SUPPORT SESSIONS****Outline of each Session**

The Basic Maths refresher will consist of **5** sessions as follows:

Session 1: fractions, decimals, percentages, rounding, ratios and proportions, order of operations (brackets), simple calculator arithmetic;

Session 2: simple algebra, simple equations, simple inequalities, using and rearranging formulae;

Session 3: simple graphs, applied problem solving and probabilities (using ‘mathematical thinking’), exponents (powers) and standard form (including calculator use);

Session 4: logarithms (including calculator use), transforming a curve to a straight line graph;

Session 5: drop-in for further practice of exercises from Sessions 1-4 (but no formal teaching / new slides).

Each session lasts for one and a half hours, although 10 minutes at the start and end of each session will be allotted for moving between classrooms.

We hope that these sessions will allow you to follow the mathematical content in some of the MSc Study Modules, such as those with ‘Statistics’ or ‘Epidemiology’ in the title. Your tutors will welcome your questions, so please do not hesitate to ask if you need assistance.

The recommended calculator is the Casio fx-85GT PLUS. You may already have a different calculator that you are comfortable with using. If so, you do not need to rush out and buy this model. However, the tutors may not be familiar with the intricate details of your calculator model!

Books such as GCSE maths exam revision guides will contain much of this material. The LSHTM Study Skills web page can provide you with further support (see <http://www.lshtm.ac.uk/edu/studyskills.html>). On this webpage you will find LSHTM’s online interactive Basic Maths self-assessment tool, and links to a number of external maths revision websites (under the subheading ‘Maths and Numeracy Skills’). You may find it helpful to have a look at these if you require some additional worked examples. They will also help to reinforce what you have done during these sessions. You can also enrol for the sessions via the interactive Basic Maths self-assessment tool.

***Aims and Objectives of Session 1***

At the end of this session you should have some basic knowledge of the following key concepts in arithmetic: fractions, decimals, percentages, rounding, ratio and proportion and order of operations. You should also have some understanding of the use of calculators in arithmetic and some indication of topics in your Term 1 modules which use these techniques.

In particular you should be able to:

- 1) add, subtract, multiply and divide fractions
- 2) convert fractions to decimals and decimals to fractions, multiply and divide decimals by multiples of 10
- 3) change a fraction to a percentage, find a percentage of a number, increase/decrease a number by a percentage
- 4) round whole numbers and decimals

- 5) use ratio and dividing in proportion
- 6) understand the order of arithmetic operations and use some basic calculator functions

***Aims and Objectives of Session 2***

At the end of this session you should have some basic knowledge of the following key concepts in algebra: algebraic expressions, simple equations, simple formulae and inequalities, and some indication of topics in your Term 1 modules which use these techniques.

In particular you should be able to:

- 1) substitute numbers for letters in algebraic expressions
- 2) multiply out brackets and use factorisation
- 3) solve simple equations
- 4) use and rearrange simple formulae
- 5) solve simple inequalities

***Aims and Objectives of Session 3***

At the end of this session you should have some basic knowledge of key concepts in graph drawing, problem solving, powers of numbers, and some indication of topics in your Term 1 modules which use these techniques.

In particular you should be able to:

- 1) understand the terminology of graphs and use axes, scales and co-ordinates
- 2) plot simple graphs
- 3) understand the equation of a straight line and use it to plot straight line graphs
- 4) understand and solve problems involving unit quantities
- 5) understand and solve problems using probability trees
- 6) use the rules for indices (multiply and divide powers, raise a power to a power, reciprocals)
- 7) understand what is meant by standard form and convert numbers to standard form

***Aims and Objectives of Session 4***

At the end of this session you should have some basic knowledge of the following key concepts in logarithms and some indication of topics in your Term 1 modules which use these techniques.

In particular you should be able to:

- 1) understand the concept of logarithms, inverse logs and natural logs
- 2) understand and use the rules of logs
- 3) use the log function on the calculator
- 4) transform non-linear to straight line graphs using logs

# Basic Maths

## Session 1: Basic Arithmetic

### Intended learning objectives

- At the end of this session you should be able to:
  - add, subtract, multiply and divide fractions
  - convert fractions to decimals and decimals to fractions, multiply and divide decimals by multiples of 10
  - change a fraction to a percentage, find a percentage of a number, increase/decrease a number by a percentage
  - round whole numbers and decimals
  - use ratio and dividing in proportion
  - understand the order of arithmetic operations and use some basic calculator functions

### § 1. Fractions (simplifying and expanding)

**numerator**  
**denominator**

If we multiply (or divide) BOTH numerator and denominator by the same number we don't change the value of the expression

▪ 'simplifying' or 'cancelling down'

$$\frac{6}{9} = \frac{2}{3} \quad (\div \text{ top and bottom by } 3)$$

▪ 'expanding'

$$\frac{3}{4} = \frac{6}{8} \quad (\times \text{ top and bottom by } 2)$$

### § 1. Fractions (multiplying and dividing)

Multiplying fractions

$$\frac{2}{7} \times \frac{3}{4} = \frac{2 \times 3}{7 \times 4} = \frac{6}{28} = \frac{3}{14}$$

Dividing fractions

$$\frac{1}{9} \div \frac{2}{5} = \frac{1}{9} \times \frac{5}{2} = \frac{5}{18}$$

- multiply numerators
- multiply denominators
- simplify

- invert second (to get the '**reciprocal**') and multiply by first

### § 1. Fractions (adding and subtracting)

$$\frac{2}{5} + \frac{1}{7} = \frac{2 \times 7}{5 \times 7} + \frac{1 \times 5}{7 \times 5} = \frac{14}{35} + \frac{5}{35} = \frac{14+5}{35} = \frac{19}{35}$$

- Find '**common denominator**'
- Expand both fractions
- Add or subtract numerators

### § 1. Fractions (rewriting)

- You may need to rewrite any mixed fractions as improper fractions BEFORE performing these operations

- '**mixed fraction**' to '**improper fraction**'

$$4\frac{1}{2} = \frac{4}{1} + \frac{1}{2} = \frac{(4 \times 2)}{(1 \times 2)} + \frac{1}{2} = \frac{(4 \times 2) + 1}{2} = \frac{8+1}{2} = \frac{9}{2}$$

- '**improper fraction**' to '**mixed fraction**'

$$\frac{7}{3} = \frac{(2 \times 3) + 1}{3} = \frac{(2 \times 3)}{3} + \frac{1}{3} = \frac{2}{1} + \frac{1}{3} = 2\frac{1}{3}$$

## § 2. Decimals

- Converting decimals to fractions:

$$0.7 = \frac{7}{10}, 4.61 = 4\frac{61}{100}, 7.949 = 7\frac{949}{1000}$$

- Convert fractions to decimals by dividing numerator by denominator:

$$\frac{1}{8} = 1 \div 8 = 0.125$$

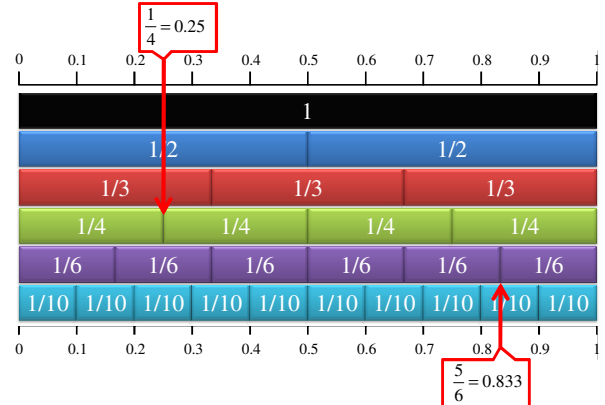
- Multiplying a decimal by a multiple of 10:

$$3.27 \times 10 = 32.7, 3.27 \times 100 = 327, 3.27 \times 1000 = 3270$$

- Dividing a decimal by a multiple of 10:

$$43.1 \div 10 = 4.31, 43.1 \div 100 = 0.431, 43.1 \div 1000 = 0.0431$$

## § 2. Fractions/Decimals in Pictures



## § 3. Percentages

- Percentages are fractions out of 100 and can be written as decimals

$$9\% = \frac{9}{100} = 0.09, 161\% = \frac{161}{100} = 1.61$$

- Converting a fraction to a percentage:

$$\frac{3}{4} \rightarrow \frac{3}{4} \times 100 = \frac{3 \times 100}{4} = \frac{3 \times 25}{1} = 75\%$$

- Percentage of a number:

$$30\% \text{ of } 60 = \frac{30}{100} \times 60 = \frac{3}{10} \times 60 = \frac{3 \times 60}{10} = \frac{3 \times 6}{1} = 18$$

- Increase/decrease a number by a percentage:

e.g. To increase 5 by 60%

$$60\% \text{ of } 5 = \frac{60}{100} \times 5 = \frac{6}{10} \times 5 = \frac{6 \times 5}{10} = \frac{6}{2} = 3, 5 + 3 = 8$$

or

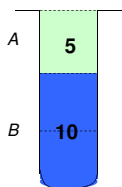
$$\frac{3 \times 5}{1} = 3, 5 + 3 = 8$$

## § 4. Rounding

Thousands Hundreds Tens Units  
7 2 8 4

- 7284 rounded to the nearest ten is 7280
- 7284 rounded to the nearest hundred is 7300
- Rounding decimals is similar e.g. 3.854 rounded to 1 decimal place is 3.9

## § 5. Ratios and proportion



- A and B are in the ratio 1:2

$$1 + 2 = 3$$

$$\frac{1}{3} \text{ of } A \text{ and } \frac{2}{3} \text{ of } B$$

- 15ml in ratio 1:2

$$\frac{1}{3} \times 15\text{ml} = \frac{15}{3}\text{ml} = 5\text{ml of } A$$

$$\frac{2}{3} \times 15\text{ml} = \frac{2 \times 15}{3}\text{ml}$$

$$= \frac{2 \times 5}{1}\text{ml} = 10\text{ml of } B$$

## § 6. Order of operations

- Do calculation from **left to right** obeying ordering:
  - Brackets (innermost 1<sup>st</sup>)
  - Exponents
  - Multiplication and Division
  - Addition and Subtraction

$$\begin{aligned} \text{E.g. } 5 + 40 \div (5 \times (12 \div 3)) &= 5 + 40 \div (5 \times 4) \\ &= 5 + 40 \div 20 \\ &= 5 + 2 \\ &= 7 \end{aligned}$$

## § 7. Applied problems

- 30ml of drug solution consists of **two thirds** drug A (costing 10p per ml), **a sixth** of drug B (costing 50p per ml) and rest of volume made up with water (no cost)
- How much does the whole solution cost?
  - A  $\rightarrow \frac{2}{3} \times 30ml = 20ml \rightarrow 20ml \times £0.10/ml = £2.00$
  - B  $\rightarrow \frac{1}{6} \times 30ml = 5ml \rightarrow 5ml \times £0.50/ml = £2.50$
  - Total cost = £2.00 + £2.50 = £4.50

- How much water is required?

$$30ml - 20ml - 5ml = 5ml \quad \text{or} \quad 1 - \frac{2}{3} - \frac{1}{6} = \frac{1}{6} \rightarrow \frac{1}{6} \times 30ml = \frac{1 \times 30}{6} ml = 5ml$$

## § 8. Topics in Term 1 modules using basic maths skills

### STATISTICS:

- Frequency
- Relative (percentage) frequency
- Arithmetic mean
- Standard deviation
- Confidence intervals
- p-values
- ...etc

### EPIDEMIOLOGY:

- Incidence
- Point prevalence (proportion)
- Risk ratio
- Rate ratio
- Odds ratio
- Mortality rates
- ...etc

## Intended learning objectives (achieved?)

- You should be able to:
  - add, subtract, multiply and divide fractions
  - convert fractions to decimals and decimals to fractions, multiply and divide decimals by multiples of 10
  - change a fraction to a percentage, find a percentage of a number, increase/decrease a number by a percentage
  - round whole numbers and decimals
  - use ratio and dividing in proportion
  - understand the order of arithmetic operations and use some basic calculator functions

## Key messages

- If we multiply or divide BOTH numerator and denominator by the same number then the value of the fraction stays the same
- To **multiply** fractions together we multiply the top numbers together and **multiply** the bottom numbers together
- To **divide** one fraction by another we invert the one we are dividing by and then **multiply** by it
- To **add and subtract** fractions we first need to rewrite the fractions so that they have the same **denominator**
- **Percentages** are fractions out of **100**

N.B. For next session: <http://www.lshtm.ac.uk/edu/studyskills.html> (subheading 'Maths and Numeracy Skills')

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**Session 1 - Basic Arithmetic Notes**

**§ 1. Fractions**

A fraction is a portion of a whole e.g.  $\frac{1}{2}$  (one half),  $\frac{1}{4}$  (one quarter),  $\frac{1}{3}$  (one third),  $\frac{5}{8}$  (five eighths). The top number is the *numerator* and the bottom number is the *denominator*.

Fractions can be *cancelled down* if we can divide both the numerator and the denominator by the same number, e.g.  $\frac{4}{6} \rightarrow \frac{2}{3}$ . We can *expand* fractions by multiplying the numerator and the denominator by the same number, e.g.  $\frac{1}{4} \rightarrow \frac{3}{12}$ .

A *mixed fraction* is a whole number and a fraction, e.g.  $1\frac{1}{2}$  (one and a half). We can also write this as a fraction  $\frac{3}{2}$ .

**To add and subtract fractions**

We need to rewrite the fractions so that they have the same denominator; we call this the *common denominator*.

e.g.  $\frac{1}{4} + \frac{2}{7}$

We can find the common denominator by multiplying together the individual denominators, in this example  $4 \times 7 = 28$ . Remember that if we change the bottom number we must also change the top number, so  $\frac{1}{4}$  becomes  $\frac{7}{28}$  and  $\frac{2}{7}$  becomes  $\frac{8}{28}$ .

Another way is to find the smallest number that can be divided by both the denominators.

e.g.  $\frac{5}{8} - \frac{1}{40} = \frac{25}{40} - \frac{1}{40} = \frac{24}{40}$ . (40 is the smallest number which can be divided by both 8 and 40). We can often cancel down the answer e.g.  $\frac{24}{40} = \frac{12}{20} = \frac{3}{5}$ .

Check if we can cancel the fractions before doing the calculation, if we can then it often makes the numbers we are working with easier to handle.

e.g.  $\frac{4}{40} + \frac{35}{70} = \frac{1}{10} + \frac{1}{2} = \frac{1}{10} + \frac{5}{10} = \frac{6}{10} = \frac{3}{5}$ .

If we have mixed fractions then one method for dealing with them is to turn them into *improper* (top heavy) fractions first,

e.g.  $1\frac{1}{2} - \frac{3}{4} = \frac{3}{2} - \frac{3}{4} = \frac{6}{4} - \frac{3}{4} = \frac{3}{4}$ .

**To multiply fractions**

Multiply the top numbers together and multiply the bottom numbers together,

e.g.  $\frac{2}{3} \times \frac{4}{7} = \frac{2 \times 4}{3 \times 7} = \frac{8}{21}$

$$1\frac{1}{2} \times \frac{3}{4} = \frac{3}{2} \times \frac{3}{4} = \frac{9}{8} = 1\frac{1}{8}$$

**To divide fractions**

When we divide one fraction by another, we multiply the first by the *reciprocal* of the second. We obtain the reciprocal by inverting (turning upside down) the fraction.

e.g.  $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$

$$\frac{3}{8} \div 1\frac{1}{2} = \frac{3}{8} \div \frac{3}{2} = \frac{3}{8} \times \frac{2}{3} = \frac{6}{24} = \frac{1}{4}$$

## § 2. Decimals

Decimals are fractions where the denominators are 10 and multiples of 10 such as 100, 1000 etc.

e.g.  $0.3 = \frac{3}{10}$ ,  $0.03 = \frac{3}{100}$ ,  $0.003 = \frac{3}{1000}$ ,  $2.3 = 2\frac{3}{10}$ ,  $23.003 = 23\frac{3}{1000}$

We convert a fraction such as  $\frac{3}{8}$  to a decimal by dividing 3 by 8 to obtain 0.375

We convert a decimal to a fraction by taking the number before the decimal point as the whole number and the numbers after the decimal point as the fraction. One decimal place gives us a fraction out of 10, 2 decimal places gives a fraction out of 100, 3 decimal places gives a fraction out of 1000 etc.

**To multiply a decimal by 10** we move the decimal point **one place to the right**.

To multiply by 100, move the decimal point two places to the right etc.

e.g.  $45.67 \times 10 = 456.7$                        $0.04 \times 100 = 4$

**To divide a decimal by 10** we move the decimal point **one place to the left**.

To divide by 100, move the decimal point two places to the left etc. Note that we put in zeroes as place markers to ensure that the decimal point is in the right place.

e.g.  $45.67 \div 10 = 4.567$                        $0.04 \div 100 = 0.0004$

## § 3. Percentages

Percentages are fractions out of 100.

e.g.  $7\%$  means 7 out of 100 =  $\frac{7}{100}$ ,  $35\% = \frac{35}{100}$ ,  $40\% = \frac{40}{100} = \frac{2}{5}$ ,  $140\% = \frac{140}{100} = \frac{7}{5}$

They can also be written as decimals.

e.g.  $7\% \rightarrow 0.07$ ,  $35\% \rightarrow 0.35$ ,  $130\% \rightarrow 1.30$

**To change a fraction to a percentage** we multiply the fraction by 100 and cancel down.

e.g.  $\frac{4}{5} \rightarrow \frac{4}{5} \times 100 = \frac{400}{5} = 80\%$

**To find a percentage of a number** we take the percentage as a fraction and **multiply** by the number.

e.g.  $20\%$  of 80 =  $\frac{20}{100} \times 80 = \frac{2}{10} \times 80 = \frac{160}{10} = 16$

$120\%$  of 80 =  $\frac{120}{100} \times 80 = \frac{12}{10} \times 80 = 96$

**To increase/decrease a number by a percentage**

Suppose we want to increase 45 by 40%

First find 40% of 45 and then add that to 45.

40% of 45 equals  $\frac{40}{100} \times 45 = \frac{4}{10} \times 45 = \frac{180}{10} = 18$ .

Then we add 18 to 45 to give 63.

If we were decreasing 45 by 40% we would subtract 18 from 45 to give 27.

Another way to increase 45 by 40% is to find 140% of 45

=  $\frac{140}{100} \times 45 = \frac{14}{10} \times 45 = \frac{630}{10} = 63$ .

Similarly to decrease 45 by 40% take 60% of 45

=  $\frac{60}{100} \times 45 = \frac{6}{10} \times 45 = \frac{270}{10} = 27$ .

## § 4. Rounding

### **Rounding whole numbers**

We can round whole numbers to the nearest ten, the nearest hundred, the nearest thousand etc.

e.g. Round 79 to the nearest 10.

7	9
tens	units

To round this to the nearest ten, take the number at the tens column which is 7, and look at the number in the next column. If the number in the next column is 5 or bigger then we round up. Here the number in the next column is 9 so we round up and the answer is 80.

(Remember when rounding we need the zero to keep the right place value).

e.g. Round 72 to the nearest 10.

7	2
tens	units

To round this to the nearest ten, take the number at the tens column which is 7, and look at the number in the next column. If the number in the next column is 5 or bigger then we round up. Here the number in the next column is 2 so we don't round up, the 7 stays the same and the answer is 70.

e.g. 712719 is seven hundred and twelve thousand, seven hundred and nineteen

7	1	2	7	1	9
hundred thousand	ten thousand	thousand	hundreds	tens	units

To round this to the nearest thousand, take the number at the thousand column which is 2, and look at the number in the next column. If the number in the next column is 5 or bigger then we round up. Here the number in the next column is 7 so we round up and the answer is 713000. Notice that we must set the zeroes in as place markers for the last three columns, since we are rounding to the nearest thousand.

### **Rounding decimals**

We can round decimals in the same way.

e.g. Round 2.718 to 1 decimal place and to 2 decimal places.

2	.	7	1	8
units		tenths	hundredths	thousandths
units		1 decimal place	2 decimal places	3 decimal places

To round this to 1 decimal place, take the number at the 1 decimal place column which is 7, and look at the number in the next column. If the number in the next column is 5 or bigger then we round up. Here the number in the next column is 1 so we don't round up, the 7 stays the same and the answer is 2.7.

To round this to 2 decimal places, take the number at the 2 decimal place column which is 1, and look at the number in the next column. If the number in the next column is 5 or bigger then we round up. Here the number in the next column is 8 so we round up, the 1 becomes a 2 and the answer is 2.72.

## § 5. Ratio and proportion

Ratio is a comparison between similar quantities. We can write a ratio as e.g. 3 to 2 or 3:2 or as a fraction  $\frac{3}{2}$ .

We write ratios using whole numbers and we write a ratio in its simplest terms

e.g.  $6:4 = 3:2$ ,  
 $12:4 = 3:1$ ,  
 $1:0.25 = 1:\frac{1}{4} = 4:1$



**Proportions**

e.g. Divide 40 in the ratio 5:3.  
 Total number of parts is  $5 + 3 = 8$ .  
 Amount of each part is  $40 \div 8 = 5$   
 Amount of first part is  $5 \times 5 = 25$   
 Amount of second part is  $3 \times 5 = 15$

So 40 divided in the ratio 5:3 is 25:15.

**Using ratio**

e.g. if two lengths are in the ratio 5:3 and the shorter length is 12 what is the longer length?

We have  $5:3 = ? : 12$  i.e.  $5/3 = ?/12$

We need to multiply the denominator, 3, by 4 to get 12. We then multiply the numerator, 5, by 4 to get 20 which is the answer.

**§ 6. Order of operations and calculator use**

**Order of operations**

Let us calculate  $24 \div 6 + 2$ .

We can work it out this way :  $24 \div 6 + 2 = 24 \div (6 + 2) = 24 \div 8 = 3$ .

Alternatively we can calculate:  $24 \div 6 + 2 = (24 \div 6) + 2 = 4 + 2 = 6$ .

Since we obtain two completely different results depending on which order we do the calculation, we can see that the order is vitally important for obtaining the required answer. Therefore, we need a set of rules to determine in which order to do the arithmetic operations.

When doing a calculation the order of operations is as follows:

- 1) **Brackets**
- 2) **Exponents** (see Session 3)
- 3) **Multiplication and Division**
- 4) **Addition and Subtraction**

In the above example, the correct order in which to do the calculation, as it is given, is:

$$24 \div 6 + 2 = (24 \div 6) + 2 = 4 + 2 = 6.$$

Note that we do the calculation from *left to right* obeying the order of operations along the way.

e.g.  $(2 \times 3) + (12 - 7) = 6 + 5 = 11$   
 $24 \div 4 + 6 \times 2 = 6 + 12 = 18$

If there are brackets within brackets, we start with the innermost bracket first.

e.g.  $5 + (10 \times (18 \div 6)) = 5 + (10 \times 3) = 5 + 30 = 35$

**Using a calculator**

To turn on the **CASIO fx-85GT PLUS** calculator, press the blue button in the top right-hand corner marked **ON**.

To turn off the **CASIO fx-85GT PLUS** calculator, press **SHIFT** (the blue button in the top left-hand corner) followed by the **AC** key (the right most key of the two orange buttons).

For basic arithmetic we need to use the **+, -, ×, ÷, =** keys. These keys can all be found in the bottom right-hand corner of the **CASIO fx-85GT PLUS** calculator. The number keys are in the bottom left-hand corner. The “(” and “)” keys are in the middle of the calculator.

For simple calculations using **CASIO fx-85GT PLUS**, press **MODE 1** to choose **COMP** from the menu list, then press **SHIFT MODE 2** to choose **Line10** (to make numbers display with decimals). The **MODE** key is next to the **ON** button.

To clear the screen, use the **AC** button. Use the orange **DEL** button to remove a mistaken entry.

Use the **SHIFT** function key if function required is in brown/green/yellow above the key. When doing more complex calculations, it is likely that you will need to use the memory function to hold the intermediate value of the calculation.

As a simple example, on **CASIO fx-85GT PLUS**, to use memory to calculate

$$(43 \times 37) + (181 - 64) =$$

key in:



to store this value (1591) in memory (i.e. using the **STO** function).

Then key in:



Your calculator should then report the answer as “1708”.

The **RCL** key is 4 keys underneath the **SHIFT** key, while the **M+** key is 4 keys underneath the **ON** key.

To clear this number (1708) from your calculator’s memory simply store the number zero in the memory by keying in:

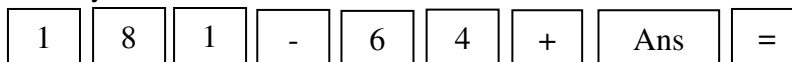


The **CASIO fx-85GT PLUS** calculator will also remember the result from the last time you pressed the **=** key in your last calculation. To recall this result you press the **Ans** key, located on the bottom row next to the **=** key. This means that, if you do the calculations above without any intermediate calculation, you can more simply key in:



to store this value (1591) temporarily in **Ans** memory.

Then key in:



Your calculator should then report the same answer as before, i.e. “1708”.

Note: Further calculator guidance is provided in the solutions to some of the questions in sessions 3 and 4 of the online interactive self-assessment (for using powers, roots, logs and inverse logs keys). The study notes for Basic Statistics for PHP, STEPH and Statistics with Computing also have sections, with examples, on using the calculator.

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**Session 1 - Basic Arithmetic Exercises**

You will **not** need a calculator for exercises 1 to 6.

**§ 1. Fractions**

1) i) Write the following fractions in their simplest form:

$$\frac{12}{18}; \quad \frac{9}{27}; \quad \frac{8}{16}; \quad \frac{19}{20}; \quad \frac{12}{20}$$

ii) Write the following as improper (top heavy) fractions:

$$1\frac{3}{4}; \quad 2\frac{2}{8}; \quad 4\frac{1}{2}; \quad 3\frac{1}{10}; \quad 5\frac{1}{3}$$

iii) Write the following as mixed fractions:

$$\frac{5}{4}; \quad \frac{3}{2}; \quad \frac{21}{8}; \quad \frac{21}{5}; \quad \frac{38}{8}$$

2) Work out the answers in their simplest form without using a calculator:

a)  $\frac{3}{4} + \frac{4}{7}$ ;    b)  $\frac{3}{4} - \frac{4}{7}$ ;    c)  $\frac{3}{4} \times \frac{4}{7}$ ;    d)  $\frac{3}{4} \div \frac{4}{7}$ ;    e)  $\frac{4}{7} \div \frac{3}{4}$

3) Work out the answers in their simplest form without using a calculator:

a)  $2\frac{1}{2} + 3\frac{3}{4}$ ;    b)  $4\frac{1}{3} - 2\frac{1}{2}$ ;    c)  $3 \times \frac{7}{8}$ ;    d)  $1\frac{3}{4} \div \frac{5}{8}$ ;    e)  $\frac{4}{7} \div 4$

**§ 2. Decimals**

1) Write the following decimals as fractions:

$$0.1 \quad 0.5 \quad 0.01 \quad 0.05 \quad 0.15 \quad 0.75 \quad 0.029$$

2) Write the following fractions as decimals:

$$\frac{14}{100}; \quad \frac{2}{1000}; \quad \frac{3}{10}; \quad \frac{4}{5}; \quad \frac{1}{4}$$

3) Write the following decimals as fractions:

$$1.25 \quad 3.5 \quad 7.4 \quad 2.04$$

4) Work out the following:

a)  $2.78 \times 100$ ;    b)  $0.03 \times 10$ ;    c)  $7.3 \div 10$ ;    d)  $0.073 \div 100$ ;    e)  $0.14 \times 100$

**§ 3. Percentages**

1) Change the following fractions to percentages:

$$\frac{3}{5}; \quad \frac{4}{8}; \quad \frac{3}{10}; \quad \frac{4}{100}; \quad \frac{1}{4}$$

2) Calculate:

$$20\% \text{ of } 180, \quad 40\% \text{ of } 60, \quad 50\% \text{ of } 90, \quad 6\% \text{ of } 550$$

3) Increase:

a) 20 by 50%    b) 50 by 20%    c) 80 by 5%

4) Decrease:

a) 20 by 40%    b) 50 by 30%    c) 80 by 45%

**§ 4. Rounding**

- 1) Round to the nearest 10:  
a) 13    b) 67    c) 49
- 2) Round to the nearest 100:  
a) 234    b) 459    c) 726
- 3) Round to 1 decimal place:  
a) 23.45    b) 45.91    c) 3.765
- 4) Round the following to the nearest ten, hundred and thousand:  
a) 45171    b) 20023    c) 1458.9
- 5) Round the following to 1, 2 and 3 decimal places:  
a) 14.5171    b) 14.5175    c) 14.5871    d) 14.5875

**§ 5. Ratio**

- 1) Write the following ratios in their simplest form:  
4:6    15:18    21:7    35:25    0.3:3
- 2) Divide:  
a) 140 in the ratio 2:3    b) 64 in the ratio 7:1    c) 130 in the ratio 2:3:5
- 3) i) Two numbers are in the ratio 4:7.  
If the smaller number is 12, what is the larger number?  
ii) Two numbers are in the ratio 5:4.  
If the larger number is 20, what is the smaller number?

**§ 6. Order of operations**

- 1) Calculate:  
a)  $2 + 6 \times 12$     b)  $(2+6) \times 12$     c)  $2 + (6 \times 12)$
- 2) Calculate:  
a)  $(20 - 8) \div (4 + 2)$     b)  $20 - 8 \div 4 + 2$
- 3) Calculate:  
 $((2 \times 3) + (16 \div 2)) - ((6 - 3) + (21 \div 7))$

**§ 7. Applied Problems**

1. There are three types of admission to a certain hospital: Ward (W), High Dependency Unit (HDU) and Intensive Care Unit (ICU). On average  $\frac{2}{12}$  of patients are admitted to HDU, and  $\frac{1}{18}$  of patients are admitted to ICU, all the rest being admitted to Ward.

- a) What fraction of patients are admitted to Ward? Express your answer in its simplest form.
- b) What is this fraction expressed as:
  - i) A decimal, rounded to 1 decimal place
  - ii) A percentage, rounded to the nearest percentage point.
- c) On average, the nightly cost for one patient is £100 in W, £500 in HDU and £1500 in ICU. Typically, 90 patients are admitted every night divided between the units in the fractions given earlier. What will be the average total nightly cost of these admissions?

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**Session 1 - Basic Arithmetic - Further Exercises**

These are intended to supplement the exercises you have already done and extend the concepts. You **will** need a calculator for some of these exercises. If you need help with using your calculator please consult your calculator manual and see the information on calculator use offered in Basic Statistics for PHP or Statistics for EPH (STEPH) or Statistics with Computing.

1) a) Put the following fractions in increasing order, smallest first (hint: convert to same denominator):

$$\frac{17}{18} \quad \frac{3}{12} \quad \frac{1}{3} \quad \frac{7}{8} \quad \frac{5}{7}$$

b) Put the following fractions in decreasing order, largest first:

$$\frac{3}{8} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{4}{5} \quad \frac{1}{10}$$

2) Work out the following:

a)  $(\frac{3}{4} \div \frac{3}{8}) + (\frac{11}{17} - \frac{2}{5})$       b)  $\frac{3}{4} \div (\frac{3}{8} + \frac{11}{17}) - \frac{2}{5}$       c)  $\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}$

3) Change the following fractions into decimals and give the answer correct to 2 decimal places:

$$\frac{5}{9} \quad \frac{7}{16} \quad \frac{20}{21} \quad \frac{14}{19} \quad \frac{4}{9}$$

4) a) What percentage is 23 out of 456? Give your answer correct to 2 decimal places.

b) What percentage is 19 out of 85? Give your answer correct to 3 decimal places.

5) a) If 12% of a number is 9.6, what is the number?

b) If 40% of a number is 22.4 what is the number?

6) a) In a town 84% of the population of 50000 receive a particular vaccination. How many people do **not** get the vaccination?

b) 30% of those who did not get the vaccine developed the disease. How many people got ill?

7) One of the following is not equal to 45%. Which one?

a)  $\frac{45}{100}$       b) 0.45      c)  $\frac{9}{20}$       d)  $\frac{45}{10}$

8) Calculate  $7\frac{5}{8} \times 1\frac{3}{4} \times 4\frac{2}{3}$ .

Give your answer both as a fraction and as a decimal correct to 1 decimal place.

9) Which of the following fractions is not equivalent to  $\frac{3}{8}$ ?

a)  $\frac{9}{24}$       b)  $\frac{6}{16}$       c)  $\frac{33}{87}$       d)  $\frac{30}{80}$

10) Express the following ratios in their lowest form:

a) 250 g : 1.25 kg      b) 35kg: 3500 g      (remember 1000 g = 1 kg)

11) a) Divide 560 in the ratio 3:7:4

b) Divide 20 in the ratio  $\frac{1}{4} : \frac{3}{4}$

12) A drugs bill is divided between two drugs J and K in the ratio of 2:3 (J: K). If the smaller share is £10,000 what is the larger share?

13)  $\frac{1}{4}$  of a drugs bill is spent on drug A.  $\frac{5}{12}$  of the bill is spent on drug B and the remainder on drug C. If the total bill is £3,890,982 how much is spent on each drug?

14) A drugs bill is split between 3 drugs X, Y, Z. If  $\frac{3}{10}$  of the total is spent on drug Y and if the amount spent on drug Y is £450,528 how much is spent on the other drugs?

15) A drugs bill is split between 3 drugs X, Y, Z in the ratio 2:3:5 (X: Y: Z).

If the amount spent on drug Y is £450,528 how much is spent on the other drugs?

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**Session 1 - Basic Arithmetic - Solutions to Exercises**

**§ 1. Fractions**

- 1) i) The fractions in their simplest form are:  
 $\frac{2}{3}; \frac{1}{3}; \frac{1}{2}; \frac{19}{20}; \frac{3}{5}$
- ii) The mixed fractions can be rewritten as:  
 $\frac{7}{4}; \frac{18}{8} = \frac{9}{4}; \frac{9}{2}; \frac{31}{10}; \frac{16}{3}$
- iii) The mixed fractions are:  
 $1\frac{1}{4}; 1\frac{1}{2}; 2\frac{5}{8}; 4\frac{1}{5}; 4\frac{6}{8} = 4\frac{3}{4}$
- 2) The answers in their simplest form are:
- a)  $\frac{3}{4} + \frac{4}{7} = \frac{21}{28} + \frac{16}{28} = \frac{37}{28} = 1\frac{9}{28}$
- b)  $\frac{3}{4} - \frac{4}{7} = \frac{21}{28} - \frac{16}{28} = \frac{5}{28}$
- c)  $\frac{3}{4} \times \frac{4}{7} = \frac{12}{28} = \frac{3}{7}$
- d)  $\frac{3}{4} \div \frac{4}{7} = \frac{3}{4} \times \frac{7}{4} = \frac{21}{16}$
- e)  $\frac{4}{7} \div \frac{3}{4} = \frac{4}{7} \times \frac{4}{3} = \frac{16}{21}$
- 3) The answers in their simplest form are:
- a)  $2\frac{1}{2} + 3\frac{3}{4} = \frac{5}{2} + \frac{15}{4} = \frac{10}{4} + \frac{15}{4} = \frac{25}{4} = 6\frac{1}{4}$
- b)  $4\frac{1}{3} - 2\frac{1}{2} = \frac{13}{3} - \frac{5}{2} = \frac{26}{6} - \frac{15}{6} = \frac{11}{6} = 1\frac{5}{6}$
- c)  $3 \times \frac{7}{8} = \frac{21}{8} = 2\frac{5}{8}$
- d)  $1\frac{3}{4} \div \frac{5}{8} = \frac{7}{4} \times \frac{8}{5} = \frac{56}{20} = \frac{14}{5} = 2\frac{4}{5}$
- e)  $\frac{4}{7} \div 4 = \frac{4}{7} \times \frac{1}{4} = \frac{1}{7}$

**§ 2. Decimals**

- 1)  $0.1 = \frac{1}{10}$        $0.5 = \frac{5}{10} = \frac{1}{2}$        $0.01 = \frac{1}{100}$        $0.05 = \frac{5}{100} = \frac{1}{20}$   
 $0.15 = \frac{15}{100} = \frac{3}{20}$        $0.75 = \frac{75}{100} = \frac{3}{4}$        $0.029 = \frac{29}{1000}$
- 2)  $\frac{14}{100} = 0.14$        $\frac{2}{1000} = 0.002$        $\frac{3}{10} = 0.3$        $\frac{4}{5} = \frac{80}{100} = 0.8$        $\frac{1}{4} = 0.25$
- 3)  $1.25 = 1\frac{1}{4}$        $3.5 = 3\frac{1}{2}$        $7.4 = 7\frac{2}{5}$        $2.04 = 2\frac{1}{25}$
- 4) a)  $2.78 \times 100 = 278$       b)  $0.03 \times 10 = 0.3$       c)  $7.3 \div 10 = 0.73$   
 d)  $0.073 \div 100 = 0.00073$       e)  $0.14 \times 100 = 14$

**§ 3. Percentages**

- 1)  $\frac{3}{5} = \frac{3}{5} \times 100 = \frac{300}{5} = 60\%$   
 $\frac{4}{8} = 50\%$        $\frac{3}{10} = 30\%$        $\frac{4}{100} = 4\%$        $\frac{1}{4} = 25\%$
- 2) 20% of 180 =  $\frac{20}{100} \times 180 = 36$   
 40% of 60 =  $\frac{40}{100} \times 60 = 24$   
 50% of 90 =  $\frac{50}{100} \times 90 = 45$   
 6% of 550 =  $\frac{6}{100} \times 550 = 33$
- 3) a)  $150/100 \times 20 = 30$       b)  $120/100 \times 50 = 60$       c)  $105/100 \times 80 = 84$

4) a)  $60/100 \times 20 = 12$       b)  $70/100 \times 50 = 35$       c)  $55/100 \times 80 = 44$

**§ 4. Rounding**

- 1) a) 10      b) 70      c) 50
- 2) a) 200      b) 500      c) 700
- 3) a) 23.5      b) 45.9      c) 3.8
- 4) a) 45170,      45200,      45000      b) 20020,      20000,      20000  
     c) 1460,      1500,      1000
- 5) a) 14.5,      14.52,      14.517      b) 14.5,      14.52,      14.518  
     c) 14.6,      14.59,      14.587      d) 14.6,      14.59,      14.588

**§ 5. Ratio**

- 1) 2:3      5:6      3:1      7:5      1:10
- 2) a)  $2+3 = 5$ ,       $140 \div 5 = 28$ ,       $28 \times 2 = 56$ ,  $28 \times 3 = 84$   
     140 divided in the ratio 2:3 is 56:84
- b)  $7+1 = 8$ ,       $64 \div 8 = 8$ ,       $8 \times 7 = 56$ ,  $8 \times 1 = 8$   
     64 divided in the ratio 7:1 is 56:8
- c)  $2+3 + 5 = 10$ ,       $130 \div 10 = 13$ ,       $13 \times 2 = 26$ ,  $13 \times 3 = 39$ ,  $13 \times 5 = 65$   
     130 divided in the ratio 2:3:5 is 26:39:65
- 3) i)  $4:7 = 12:?$ ,       $4 \times 3 = 12$  so  $7 \times 3 = 21$  and the ratio becomes  $4:7 = 12: 21$ .  
     The larger number is 21.
- ii)  $5:4 = 20:?$        $5 \times 4 = 20$  so  $4 \times 4 = 16$  and the ratio becomes  $5:4 = 20:16$ .  
     The smaller number is 16.

**§ 6. Order of Operations**

- 1) a)  $2 + 6 \times 12 = 2 + 72 = 74$       b)  $(2+6) \times 12 = 8 \times 12 = 96$   
     c)  $2 + (6 \times 12) = 2 + 72 = 74$
- 2) a)  $(20 - 8) \div (4 + 2) = 12 \div 6 = 2$       b)  $20 - 8 \div 4 + 2 = 20 - 2 + 2 = 20$
- 3)  $((2 \times 3) + (16 \div 2)) - ((6 - 3) + (21 \div 7)) = (6 + 8) - (3 + 3) = 14 - 6 = 8$

**§ 7. Applied Problems**

- 1) a) Fraction admitted to HDU and ICU is  $2/12 + 1/18 = 1/6 + 1/18 = 3/18 + 1/18 = 4/18$ . So fraction admitted to W is  $1 - 4/18 = 14/18 = 7/9$ .
- b)  $7/9 = 0.7777... =$  i) 0.8 to 1 decimal place ii) 78% to nearest % point.
- c)  $1/6 \times 90 = 15$  HDU patients;  $1/18 \times 90 = 5$  ICU patients; remainder = 70 W patients.  
     So typical nightly cost is  
      $(15 \times 500) + (5 \times 1500) + (70 \times 100) = 7500 + 7500 + 7000 = \pounds 22000$ .

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**Session 1 - Basic Arithmetic Further Exercises - Solutions**

- 1) a) Increasing order, smallest first:  $\frac{3}{12}$   $\frac{1}{3}$   $\frac{5}{7}$   $\frac{7}{8}$   $\frac{17}{18}$   
 b) Decreasing order, largest first:  $\frac{4}{5}$   $\frac{2}{3}$   $\frac{1}{2}$   $\frac{3}{8}$   $\frac{1}{10}$

- 2) a)  $(\frac{3}{4} \times \frac{8}{3}) + (\frac{55-34}{85}) = \frac{24}{12} + \frac{21}{85} = 2 + \frac{21}{85} = 2\frac{21}{85}$   
 b)  $\frac{3}{4} \div (\frac{3}{8} + \frac{11}{17}) - \frac{2}{5} = \frac{3}{4} \div (\frac{51}{136} + \frac{88}{136}) - \frac{2}{5} = \frac{3}{4} \div \frac{139}{136} - \frac{2}{5}$   
 $= \frac{3}{4} \times \frac{136}{139} - \frac{2}{5} = \frac{102}{139} - \frac{2}{5} = \frac{510}{695} - \frac{278}{695} = \frac{232}{695}$   
 c)  $\frac{1}{2}$

- 3) 0.56 0.44 0.95 0.74 0.44

- 4) a)  $\frac{23}{456} \times 100 = 5.04$  (converting the fraction  $\frac{23}{456}$  to a decimal)  
 b)  $\frac{19}{85} \times 100 = 22.353$  (converting the fraction  $\frac{19}{85}$  to a decimal)

- 5) a) 12% is 9.6 so 1% is  $\frac{9.6}{12}$  and 100% is  $\frac{9.6}{12} \times 100 = 80$   
 b) 40% is 22.4 so 1% is  $\frac{22.4}{40}$  and 100% is  $\frac{22.4}{40} \times 100 = 56$

- 6) a) 16% do not get the vaccine.  $\frac{16}{100} \times 50000 = 8000$   
 b) 30% of 8000 get ill.  $\frac{30}{100} \times 8000 = 2400$

7) d) is not equal to 45%

- 8)  $7^5/8 \times 1^3/4 \times 4^2/3 = \frac{61}{8} \times \frac{7}{4} \times \frac{14}{3} = \frac{61 \times 7 \times 14}{8 \times 4 \times 3} = \frac{5978}{96} = \frac{2989}{48} = 62.3$

- 9) c)  $\frac{33}{87}$  does not equal  $\frac{3}{8}$

- 10) a) 250 g : 1.25 kg = 250 g : 1250g = 25:125 = 1:5  
 b) 35kg: 3500 g = 35000 g : 3500 g = 350:35 = 10:1

- 11) a) ratio 3:7:4 so take 3+7+4 = 14  
 find  $\frac{3}{14}$ ,  $\frac{7}{14}$  and  $\frac{4}{14}$  of 560 which is  $\frac{3}{14} \times 560$ ,  $\frac{7}{14} \times 560$  and  $\frac{4}{14} \times 560$   
 so 560 divided in the ratio 3:7:4 is 120:280:160  
 b) ratio  $\frac{1}{4} : \frac{3}{4}$  is same as ratio 1:3  
 $\frac{1}{4} \times 20 = 5$   $\frac{3}{4} \times 20 = 15$  so 20 divided in the ratio  $\frac{1}{4} : \frac{3}{4}$  is 5:15

- 12)  $\frac{2}{3} = \frac{10000}{?}$  So the larger share is given by  $\frac{3}{2} \times 10000 = 15000$

- 13)  $\frac{1}{4} \times 3,890,982 = \text{£}972745.50$   $\frac{5}{12} \times 3,890,982 = \text{£}1621242.50$   
 $\text{£}3,890,982 - (\text{£}972745.50 + \text{£}1621242.50) = \text{£}1296994$

- 14)  $\frac{3}{10}$  of the total is £450,528 (the amount spent on drug Y)  
 so  $\frac{1}{10}$  of the total is  $\frac{\text{£}450,528}{3}$   
 so  $\frac{10}{10}$  (the whole) of the total is  $\frac{\text{£}450,528}{3} \times 10 = \text{£}1501760$   
 so the amount spent on the other drugs is  $1501760 - 450,528 = \text{£}1051232$

- 15) Splitting the bill according to the ratio 2:3:5 is equivalent to taking  $\frac{2}{10}$ ,  $\frac{3}{10}$ ,  $\frac{5}{10}$  so  
 $\frac{3}{10}$  of the total is £450,528 (the amount spent on drug Y) and this is the same as question 14.



# Basic Maths

## Session 2: Basic Algebra

### Intended learning objectives

- At the end of this session you should be able to:
  - substitute numbers for letters in algebraic expressions
  - multiply out brackets and use factorisation
  - solve simple equations
  - use and rearrange simple formulae
  - solve simple inequalities

#### § 1. Algebraic expressions (indices and roots)

$$3 \times 3 \times 3 \times 3 = 3^4$$

1
2
3
4

← 'index' 'power' 'exponent'  
← 'base'

$n \times n = n^2$  'n squared' or 'n to the power 2'

$n \times n \times n = n^3$  'n cubed' or 'n to the power 3'

$n \times n \times n \times n = n^4$  'n to the power 4'

Roots can be used to undo indices:

Square root:  $\sqrt[n^2]{n^2} = n$ , (usually written as  $\sqrt{n^2} = n$ )

Cube root:  $\sqrt[3]{n^3} = n$

Fourth root:  $\sqrt[4]{n^4} = n$ , and so on

#### § 1. Algebraic expressions (substitution, + - × ÷ terms)

▪ **'Substitution':** If  $x = 3$  and  $y = 6$   
 $5x - 2y = (5 \times 3) - (2 \times 6) = 15 - 12 = 3$

▪ Adding and subtracting like terms:

$$6a + 4b - a + 7b = (6 - 1)a + (4 + 7)b = 5a + 11b$$

▪ Multiplying and dividing algebraic terms:

$$\frac{2p^3 \times 5q}{4p} = \frac{2 \times p \times p \times p \times 5 \times q}{4 \times p} = \frac{(2 \times 5) \times p \times p \times q}{4} = \frac{10p^2q}{4} = \frac{5p^2q}{2}$$

▪ Algebraic fractions:

$$\frac{3}{x} - \frac{9}{4y} = \frac{(3 \times 4y) - (9 \times x)}{4xy} = \frac{12y - 9x}{4xy}$$

#### § 1. Algebraic expressions (multiplying out, factorisation)

▪ Multiplying out brackets:

$$3(x + 2y) = 3 \times (x + 2y) = 3 \times x + 3 \times 2 \times y = 3x + 6y$$

$$-2(5x + y) = -2 \times (5x + y) = (-2) \times 5 \times x + (-2) \times y = -10x - 2y$$

$$(2x + y)(3x + 4y) = 2x \times 3x + 2x \times 4y + y \times 3x + y \times 4y$$

$$= 6x^2 + 8xy + 3xy + 4y^2 = 6x^2 + 11xy + 4y^2$$

▪ Factorisation:

$$3x - 6y = 3(x - 2y)$$

$$3x + xy - 2xz = x(3 + y - 2z)$$

#### § 2. Simple equations (solving)

$$-\frac{2}{3}x + \frac{4}{5} = \frac{1}{3}x - 2$$

Find  $x$ :

$$\frac{4}{5} = \frac{1}{3}x + \frac{2}{3}x - 2$$

$$2 + \frac{4}{5} = \frac{1}{3}x + \frac{2}{3}x$$

$$\frac{10 + 4}{5} = \left(\frac{1 + 2}{3}\right)x$$

$$\frac{14}{5} = x$$

### § 3. Formulae (basics)

- A formula is an equation that describes the relationship between two or more quantities
- Suppose  $Q = 1.4P + 3$
- If  $P = 2$   
 $Q = 1.4 \times 2 + 3 = 2.8 + 3 = 5.8$

### § 3. Formulae (rearranging)

- Rearrange this formula to make  $P$  the subject:

$$T = \sqrt[3]{\frac{4P}{P-Q}}$$

$$T^3 = \frac{4P}{P-Q}$$

$$T^3(P-Q) = 4P$$

$$T^3P - T^3Q = 4P$$

$$T^3P - 4P = T^3Q$$

$$P(T^3 - 4) = T^3Q$$

$$P = \frac{T^3Q}{T^3 - 4}$$

### § 4. Simple inequalities ( $><\geq\leq$ )

- Greater than:  $>$
- Less than:  $<$
- Greater than or equal to:  $\geq$
- Less than or equal to:  $\leq$

### § 4. Simple inequalities (solving)

$$3x - 5 \geq 7x + 8$$

$$3x - 7x \geq 8 + 5$$

$$-4x \geq 13$$

$$x \leq \frac{13}{-4}$$

(note inequality sign change when  $\div$  by negative number)

$$x \leq -\frac{13}{4}$$

### § 5. Applied problems

- Suppose there are  $N$  people of which  $I$  are infected with some disease and the rest are susceptible ( $S$ )
- Write the formula connecting  $N$ ,  $I$  and  $S$ , with  $N$  as the subject
  - $> N = I + S$
- What proportion ( $p$ ) of people are infected?
  - $> p = \frac{I}{N}$
- Write  $p$  in terms of  $S$  and  $N$ 
  - $> p = \frac{N-S}{N}$

### § 5. Applied problems (cont.)

- Make  $N$  the subject of this formula for  $p$

$$p = \frac{N-S}{N}$$

$$Np = N - S$$

$$Np - N = -S$$

$$N(p-1) = -S$$

$$N = \frac{-S}{(p-1)}$$

$$N = \frac{S}{(1-p)}$$

or

$$Np + S = N$$

$$S = N - Np$$

$$S = N(1-p)$$

$$\frac{S}{(1-p)} = N$$

Note that  $p$  is a proportion so  $0 \leq p \leq 1$  and  $(p-1) \leq 0$

## § 6. Topics in Term 1 modules using basic maths skills

### **Formulae**

- Calculating test statistics (e.g. z-test using formula for standard error)
- Calculating confidence intervals
- Calculating correlation coefficient
- Standardised mortality ratios

### **Inequalities**

- Categorising variables
- Determining significance using p-values

## Intended learning objectives (achieved?)

- You should be able to:
  - substitute numbers for letters in algebraic expressions
  - multiply out brackets and use factorisation
  - solve simple equations
  - use and rearrange simple formulae
  - solve simple inequalities

## Key messages

- Algebra is about making **letters** represent quantities
- We can add and **subtract** like terms
- We can multiply and **divide** algebraic terms
- **Factorisation** is the reverse of multiplying out brackets
- To solve a simple equation or **inequality** we need to find the value of the unknown quantity which is represented by the letter
- To rearrange a formula:  
Remove roots; clear fractions and **brackets**; collect terms involving the required subject; factorise if necessary; isolate the required subject

N.B. For next session: <http://www.lshtm.ac.uk/edu/studyskills.html>  
(subheading 'Maths and Numeracy Skills')

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**Session 2 - Basic Algebra Notes**

**§ 1. Algebraic Expressions**

In algebra we use letters and symbols as well as numbers to represent quantities e.g. instead of saying that the cost of a drug is £25 we could say let the cost of the drug be £P where P represents a number.

**Substitution** e.g. if  $x = 2$  and  $y = 5$ , find the values of i)  $3 + x$ , ii)  $y - x$ , iii)  $2y - 7$  iv)  $y/x$ , v)  $\frac{5x + 4y}{2y + 5}$

i)  $3 + x = 3 + 2 = 5$

ii)  $y - x = 5 - 2 = 3$

iii)  $2y - 7 = 2 \times 5 - 7 = 10 - 7 = 3$

(note that multiplication signs are often missed out in algebra, here  $2y$  means  $2 \times y$ )

iv)  $y/x = 5/2 = 2\frac{1}{2}$

v)  $\frac{5x + 4y}{2y + 5} = \frac{(5 \times 2) + (4 \times 5)}{(2 \times 5) + 5} = (10 + 20) / (10 + 5) = 30/15 = 2$

**Adding and subtracting like terms**

e.g.  $2x + 3x + 7x = (2+3+7)x = 12x$   
 $3z^2 + 10z^2 - 4z^2 = (3 + 10 - 4)z^2 = 9z^2$   
 $5a + 10b - 3a + 4b + c = 5a - 3a + 10b + 4b + c = 2a + 14b + c$

**Multiplying and dividing algebraic terms**

e.g.  $5x \times 2y = 5 \times x \times 2 \times y = 5 \times 2 \times x \times y = 10xy$   
 $-5 \times x = -5x$  which is the same as  $5 \times -x = -5x$   
 $-5x \times 2y = -5 \times x \times 2 \times y = -5 \times 2 \times x \times y = -10xy$   
 $5x \div 10y = \frac{5x}{10y} = \frac{x}{2y}$   
 $5x \div -10y = \frac{5x}{-10y} = -\frac{x}{2y}$  (we can cancel down the fractions)  
 $3m \times 4m \times 2p = 3 \times 4 \times 2 \times m \times m \times p = 24 \times m^2 \times p = 24m^2p$   
 (we are using indices to simplify our answer)  
 $\frac{3q^2 \times 2p}{4q} = \frac{3 \times \cancel{q} \times q \times 2 \times p}{4 \times \cancel{q}} = \frac{6qp}{4} = \frac{3qp}{2}$  (we can also cancel letters)

**Algebraic fractions**

We use the same rules as for ordinary fractions.

e.g.  $\frac{y}{2} + \frac{y}{3} = \frac{3y}{6} + \frac{2y}{6} = \frac{3y + 2y}{6} = \frac{5y}{6}$   
 $\frac{5}{x} + \frac{6}{y} = \frac{5y}{xy} + \frac{6x}{xy} = \frac{5y + 6x}{xy}$   
 $\frac{5}{x} - \frac{6}{2x} = \frac{10}{2x} - \frac{6}{2x} = \frac{10 - 6}{2x} = \frac{4}{2x} = \frac{2}{x}$

$$\frac{5}{x} \times \frac{6}{2x} = \frac{30}{2x^2} = \frac{15}{x^2}$$

$$\frac{5}{x} \div \frac{6}{2xy} = \frac{5}{x} \times \frac{2xy}{6} = \frac{10y}{6} = \frac{5y}{3}$$

**Multiplying out brackets**

Brackets are used to group expressions together. When removing brackets, each expression in the bracket is multiplied by the quantity outside the bracket.

e.g.  $2(x + 3y) = 2 \times (x + 3y) = 2 \times x + 2 \times 3 \times y = 2x + 6y$   
 $2(x - 3y) = 2 \times (x - 3y) = 2 \times x - 2 \times 3 \times y = 2x - 6y$

If there is a minus sign outside the bracket, the signs inside the bracket are changed when the bracket is removed.

e.g.  $-2(x + y) = -2x - 2y$   
 $-3(3a - 2b) = -9a + 6b$

[Why does this happen?  $-2(x + y) = -2 \times (x + y) = (-2) \times x + (-2) \times y = -2x + -2y = -2x - 2y$ ]

We can have more than one set of brackets.

e.g.  $2(x + 3y) + 4(3x - y) = 2x + 6y + 12x - 4y = 2x + 12x + 6y - 4y = 14x + 2y$   
 $2(x + 3y) - 4(3x - y) = 2x + 6y - 12x + 4y = 2x - 12x + 6y + 4y = -10x + 10y$   
 $x(4 + 3y) - x(3 - y) = 4x + 3xy - 3x + xy = 4x - 3x + 3xy + xy = x + 4xy$

We can have 2 brackets multiplied together:

e.g.  $(x + 2y)(2x + 3y) = x \times 2x + x \times 3y + 2y \times 2x + 2y \times 3y = 2x^2 + 3xy + 4xy + 6y^2 = 2x^2 + 7xy + 6y^2$

**Factorisation**

This is the opposite of removing brackets – here we are trying to find common factors to pull out of the expression and leave the rest in brackets.

e.g.  $2x + 6y = 2(x + 3y)$

2 is a common factor of 2x and 6y so we take it outside the bracket. We divide 2x and 6y by 2 and whatever is left goes in the bracket.

e.g.  $3x + 12xy = 3x(1 + 4y)$   
 $6x^2 + 12x = 6x(x + 2)$

$4(x-y) + 2b(x-y) = (x-y)(4 + 2b)$  here (x-y) is the common factor

**§ 2. Simple Equations**

$4x + 3 = 15$  is an example of a simple equation.

To **solve the equation** we need to find the value of the unknown quantity which is represented by the letter x, which will make the left hand side of the equation equal to the right hand side of the equation.

$4x + 3 = 2x + 9$  is also an example of a simple equation. Here there are unknown quantities (4x and 2x) written on both sides of the equation.

The strategy is always to try to get the *unknowns* (or *variables*) onto one side of the equals sign and to get the *numbers* (or *constant* terms) onto the other side.

Remember that whatever is done to one side of an equation must be done to the other side.

e.g.  $4x + 3 = 15$  (subtract 3 from both sides)  
 $4x = 12$  (divide both sides by 4)  
 $x = 3$

e.g.  $4x - 3 = 2x + 3$  (subtract  $2x$  from both sides)  
 $4x - 2x = 3 + 3$  (and add 3 to both sides)  
 $2x = 6$  (divide both sides by 2)  
 $x = 3$

e.g.  $\frac{y}{3} + \frac{y}{5} = \frac{5}{6}$   
 $\frac{5y + 3y}{15} = \frac{5}{6}$   
 $\frac{8y}{15} = \frac{5}{6}$  (multiply both sides by 15 and by 6 to clear the fraction. Effectively we are cross-multiplying).  
 $48y = 75$  therefore  $y = 75/48 = 25/16$

### § 3. Formulae

A formula is an equation that describes the relationship between two or more quantities, such as

$$F = 1.8C + 32$$

where  $F$  is the **subject of the formula**.

e.g. In the above formula, if  $C = 100$  what is the value of  $F$ ?

$$F = 1.8 \times 100 + 32 = 180 + 32 = 212.$$

We may have to **rearrange the formula** to make one of the other quantities the subject of the formula.

e.g. In the above formula, if  $F = 32$ , what is the value of  $C$ ?

$$F = 1.8C + 32$$

$$F - 32 = 1.8C$$

$$\frac{F - 32}{1.8} = C$$

$$\text{If } F = 32 \text{ then } \frac{32 - 32}{1.8} = 0, \text{ i.e. } C = 0.$$

#### **To rearrange a formula**

Remove square roots or other roots, clear fractions, clear brackets, collect terms involving the required subject of the formula, factorise if necessary, isolate the required subject.

e.g. Make  $S$  the subject of the formula  $T = \sqrt{\frac{2S}{S-v}}$

Clear square root	$T^2 = \frac{2S}{S-v}$
Clear fraction	$T^2(S-v) = 2S$
Clear bracket	$T^2S - T^2v = 2S$
Collect terms in $S$	$T^2S - 2S = T^2v$
Factorise	$S(T^2 - 2) = T^2v$
Isolate $S$	$S = \frac{T^2v}{T^2 - 2}$

This example uses all the steps – you won't necessarily have to do all this every time!

**§ 4. Simple Inequalities**

The following symbols are used for inequalities:

$>$	$\geq$	$<$	$\leq$
Greater than	Greater than or equal to	Less than	Less than or equal to

**The rules for solving inequalities are the same as for equations except if multiplying or dividing both sides of the inequality by a negative number. This reverses the inequality sign.**

(Why does this happen? Think about this: we know that 8 is bigger than 4, i.e.  $8 > 4$ ; now multiply both sides by  $-1$ ;  $-8$  is not bigger than  $-4$ , it is smaller than  $-4$ ; i.e.  $-8 < -4$ )

e.g. Solve the inequality  $4x + 7 \geq 2x + 10$ .

$$\begin{aligned}
 4x + 7 &\geq 2x + 10 \\
 4x - 2x &\geq 10 - 7 \\
 2x &\geq 3 \\
 x &\geq 3/2
 \end{aligned}$$

e.g. Solve  $2x - 8 \geq 4x + 10$

$$\begin{aligned}
 2x - 8 &\geq 4x + 10 \\
 2x - 4x &\geq 10 + 8 \\
 -2x &\geq 18 \\
 x &\leq -18/2 \text{ i.e. } x \leq -9
 \end{aligned}$$

e.g. Solve  $10/x + 5 > 25$  (note here that  $x$  cannot be zero – we cannot divide by zero - and  $x$  cannot be negative)

$$\begin{aligned}
 10 + 5x &> 25x \\
 10 &> 25x - 5x \\
 10 &> 20x \text{ therefore } 10/20 > x \text{ and } 1/2 > x \text{ which is the same as } x < 1/2. \\
 \text{However, since } x &\text{ cannot be negative the full solution is } 0 < x < 1/2. \\
 \text{(Try putting } x = -1, &\text{ then } 10/_{-1} + 5 \text{ is } -10 + 5 \text{ which is } -5, \text{ which is not greater than } 25).
 \end{aligned}$$

**London School of Hygiene & Tropical Medicine  
BASIC MATHS SUPPORT SESSIONS****Session 2 - Basic Algebra Exercises**

You do **not** need to use a calculator for these exercises.

**§ 1. Algebraic Expressions**

1) If  $x = 3$  and  $y = 4$ , find the values of:

i)  $y - x$ ,      ii)  $3y - 5$       iii)  $\frac{9y - 6}{2x}$

2) Simplify the following:

i)  $2y^2 + 10y^2 - 3y^2$       ii)  $9x + 10z - 3x + z$       iii)  $9x \times 10x \div 3y$

3) Multiply out the following brackets:

i)  $3(2z - 4)$       ii)  $-3(2z - 4)$       iii)  $4(2z + 3) - 3(z - 2)$

4) Factorise the following expressions:

i)  $8y + 18$       ii)  $14z - 7y$       iii)  $9x^2 + 6x$

**§ 2. Simple Equations**

1) Solve the following equations:

i)  $9x + 3 = 39$       ii)  $8y - 4 = 5y + 8$       iii)  $18 - 3x = 3$       iv)  $4x + 7 = 52 - 5x$

2) Solve the following equations:

i)  $\frac{2y}{5} = \frac{y}{8} + \frac{1}{2}$       ii)  $\frac{x}{4} + \frac{x}{5} = \frac{9}{2}$

**§ 3. Formulae**

If  $p = 4$  and  $q = 6$  find the value of  $T$  in the following formulae:

i)  $p = 3T/q$       ii)  $q = 3(T - p)$       iii)  $T^2 = pq + 1$

**§ 4. Simple Inequalities**

Solve the following inequalities:

i)  $4y + 7 \geq 2y + 11$       ii)  $3y + 24 < 8y - 11$       iii)  $-4x + 12 > 20$



**§ 5. Applied problems**

1. Patients diagnosed with a certain medical condition have a risk  $p$  of dying within one year of diagnosis:  $p$  is simply the average proportion of diagnosed patients who die within a year. Let  $n$  be the number of diagnosed patients, and  $k$  be the average number of these who die within a year.

a) Express  $p$  in terms of  $k$  and  $n$ .

b) Express  $k$  in terms of  $p$  and  $n$ .

c) Suppose that we define  $w = \frac{k}{n-k}$ . (In fact, defined this way,  $w$  is called the *odds* of dying.) Using your answer in b) to substitute for  $k$ , express  $w$  in terms of  $n$  and  $p$ .

d) By factorising the denominator (bottom of fraction) in your answer to c), express  $w$  in terms of  $p$ .

e) If you've got this far, use your answer to d) to express  $p$ , the risk, in terms of  $w$ , the odds.

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BASIC MATHS SUPPORT SESSIONS****Session 2 - Basic Algebra Further Exercises**

These are intended to supplement the exercises you have already done and extend the concepts.

You **will** need a calculator for some of these exercises. If you need help with using your calculator please consult your calculator manual and see the information on calculator use offered in Basic Statistics for PHP or Statistics for EPH (STEPH) or Statistics with Computing.

1) Factorise the following expressions:

i)  $8x^3 - 4x^2$     ii)  $xa^2 - ya$     iii)  $x(a - b) + 3(a - b)$     iv)  $4(p + q) - r(p + q)$     v)  $6f + 12g - 18h$

2) Solve the following equations:

a)  $5(x - 3) = 20$     b)  $7(2-3y) = 3(5y-1)$     c)  $\frac{z+3}{2} = \frac{z-3}{3}$

3) Make t the subject of the following formulae:

i)  $v = u + at$     ii)  $x = \frac{1}{2}at^2$     iii)  $C = \frac{n-t}{3r}$     iv)  $S = pq(t + q)$

4) Rewrite the following making the given letter the subject of the formula:

i)  $r = \sqrt{\frac{Z}{4g}}$  Z the subject    ii)  $q = \frac{p}{\sqrt{r}}$  r the subject    iii)  $y = \frac{3-2x}{4x+7}$  x the subject

5) If  $H - mg = mb$ , find:

- a) the value of H when  $m=4$ ,  $g=10$ ,  $b=6$
- b) the value of g when  $H = 30$ ,  $m=3$ ,  $b=4$
- c) the value of b when  $H=65$ ,  $m=6$ ,  $g=10$

6) Solve the following inequalities:

i)  $y + 3 \geq 9 + 3y$     ii)  $7(2-3y) < 3(5y-1)$     iii)  $\frac{x}{2} > \frac{x}{5} + 1$

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**BASIC MATHS SUPPORT SESSIONS**

**Session 2 - Basic Algebra Solutions**

**§ 1. Algebraic Expressions**

1) i)  $y - x = 4 - 3 = 1$     ii)  $3y - 5 = 3 \times 4 - 5 = 12 - 5 = 7$     iii)  $\frac{9y - 6}{2x} = \frac{36 - 6}{6} = \frac{30}{6} = 5$

2) i)  $2y^2 + 10y^2 - 3y^2 = (2 + 10 - 3)y^2 = 9y^2$   
 ii)  $9x + 10z - 3x + z = 9x - 3x + 10z + z = 6x + 11z$   
 iii)  $9x \times 10x \div 3y = 90x^2/3y = 30x^2/y$

3) i)  $6z - 12$     ii)  $-6z + 12$     iii)  $4(2z + 3) - 3(z - 2) = 8z + 12 - 3z + 6 = 5z + 18$

4) i)  $8y + 18 = 2(4y + 9)$     ii)  $14z - 7y = 7(2z - y)$     iii)  $9x^2 + 6x = 3x(3x + 2)$

**§ 2. Simple Equations**

1) i)  $9x + 3 = 39$  so  $9x = 39 - 3$ ;  $9x = 36$ ;  $x = 36/9$ ;  $x = 4$   
 ii)  $8y - 4 = 5y + 8$  so  $8y - 5y = 8 + 4$ ;  $3y = 12$ ;  $y = 12/3$ ;  $y = 4$   
 iii)  $18 - 3x = 3$  so  $-3x = 3 - 18$ ;  $-3x = -15$ ;  $x = -15/-3$ ;  $x = 5$   
 (or  $18 - 3 = 3x$ ;  $15 = 3x$ ;  $15/3 = x$ ; and  $x = 5$ )  
 iv)  $4x + 7 = 52 - 5x$  so  $4x + 5x = 52 - 7$ ;  $9x = 45$ ;  $x = 45/9$ ;  $x = 5$

2) i)  $\frac{2y}{5} = \frac{y}{8} + \frac{1}{2}$     so  $\frac{2y}{5} - \frac{y}{8} = \frac{1}{2}$  and  $\frac{16y - 5y}{40} = \frac{1}{2}$ ;  $\frac{11y}{40} = \frac{1}{2}$ ;  $11y = \frac{40}{2}$ ;  $11y = 20$ ;  $y = \frac{20}{11}$   
 ii)  $\frac{x}{4} + \frac{x}{5} = \frac{9}{2}$  so  $\frac{5x + 4x}{20} = \frac{9}{2}$ ;  $\frac{9x}{20} = \frac{9}{2}$ ;  $9x = \frac{20 \times 9}{2} = 90$ ;  $x = 90/9$ ;  $x = 10$

**§ 3. Formulae**

$p = 4$  and  $q = 6$

i)  $p = 3T/q$  so  $3T = pq$  and  $T = pq/3 = 4 \times 6/3 = 8$   
 ii)  $q = 3(T - p)$  so  $T = q/3 + p$  and  $T = 6/3 + 4 = 6$   
 iii)  $T^2 = pq + 1$  so  $T = \sqrt{pq + 1}$  and  $T = \sqrt{4 \times 6 + 1} = \sqrt{25} = 5$

**§ 4. Simple Inequalities**

i)  $4y + 7 \geq 2y + 11$     so  $4y - 2y \geq 11 - 7$ ;  $2y \geq 4$ ;  $y \geq 2$   
 ii)  $3y + 24 < 8y - 11$     so  $24 + 11 < 8y - 3y$ ;  $35 < 5y$ ;  $35/5 < y$ ;  $7 < y$  which is the same as  $y > 7$   
 iii)  $-4x + 12 > 20$  so  $-4x > 20 - 12$ ;  $-4x > 8$  and  $x < -8/4$ ;  $x < -2$  (reverse the sign)

**§ 5. Applied problems**

- 1) a)  $p = k/n$                       b)  $k = np$                       c)  $w = \frac{np}{n-np}$
- d)  $w = \frac{np}{n(1-p)} = \frac{p}{1-p}$ , since  $n$  cancels in top and bottom of fraction
- e)
- $$w = \frac{p}{1-p} \Rightarrow p = w(1-p) = w - wp$$
- $$\therefore p + wp = w \Rightarrow p(1+w) = w$$
- $$\therefore p = \frac{w}{1+w}$$

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**Session 2 - Basic Algebra Further Exercises – Solutions**

1) i)  $4x^2(2x - 1)$  ii)  $a(xa - y)$  iii)  $(a - b)(x + 3)$  iv)  $(p + q)(4 - r)$  v)  $6(f + 2g - 3h)$

2) a)  $5x - 15 = 20$  so  $5x = 20 + 15 = 35$  and  $x = 7$

b)  $14 - 21y = 15y - 3$  so  $14 + 3 = 15y + 21y$ ;  $17 = 36y$  and  $y = 17/36$

c)  $3(z + 3) = 2(z - 3)$  so  $3z + 9 = 2z - 6$ ;  $3z - 2z = -6 - 9$ ;  $z = -15$

3) i)  $at = v - u$  so  $t = \frac{v - u}{a}$  ii)  $2x = at^2$  so  $t^2 = \frac{2x}{a}$ ;  $t = \sqrt{\frac{2x}{a}}$

iii)  $3rC = n - t$  so  $t = n - 3rC$  iv)  $t + q = \frac{S}{pq}$  so  $t = \frac{S}{pq} - q$

4) i)  $r^2 = \frac{Z}{4g}$ ;  $Z = r^2 4g$  ii)  $q\sqrt{r} = p$  so  $\sqrt{r} = p/q$  and  $r = (p/q)^2$

iii)  $4xy + 7y = 3 - 2x$  and  $4xy + 2x = 3 - 7y$  so  $2x(2y+1) = 3 - 7y$  and  $2x = \frac{3 - 7y}{2y + 1}$

so  $x = \frac{3 - 7y}{2(2y + 1)}$

5) a)  $H = mb + mg = 4 \times 6 + 4 \times 10 = 24 + 40 = 64$

b)  $g = (H - mb)/m = (30 - 3 \times 4)/3 = (30 - 12)/3 = 18/3 = 6$

c)  $b = (H - mg)/m = (65 - 6 \times 10)/6 = (65 - 60)/6 = 5/6$

6) i)  $3 - 9 \geq 3y - y$  so  $-6 \geq 2y$  and  $-6/2 \geq y$  or  $y \leq -3$

ii)  $14 - 21y < 15y - 3$  so  $14 + 3 < 15y + 21y$ ;  $17 < 36y$  and  $17/36 < y$  or  $y > 17/36$

iii)  $x/2 - x/5 > 1$  so  $(5x - 2x)/10 > 1$  and  $3x/10 > 1$ ;  $3x > 10$ ;  $x > 10/3$

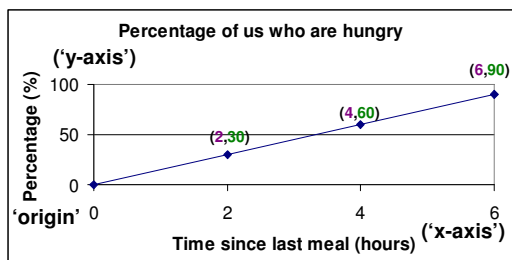
# Basic Maths

## Session 3: Graphs, Problem Solving, and Powers

### Intended learning objectives

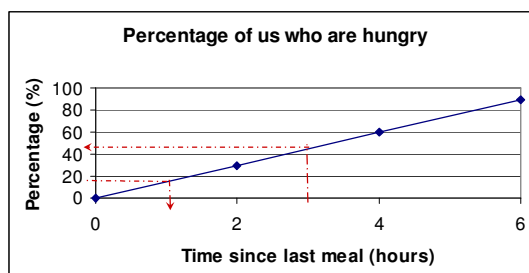
- At the end of this session you should be able to:
  - understand the terminology of graphs and use axes, scales and co-ordinates
  - plot simple graphs
  - understand the equation of a straight line and use it to plot straight line graphs
  - understand and solve problems involving unit quantities
  - understand and solve problems using probability trees
  - use the rules for indices (multiply and divide powers, raise a power to a power, reciprocals)
  - understand what is meant by standard form and convert numbers to standard form

### § 1. Plotting graphs (basics)



Time since last meal (hours)	0	2	4	6
Percentage of us hungry (%)	0	30	60	90

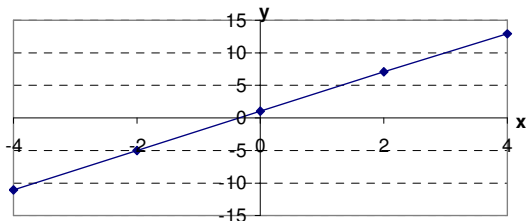
### § 1. Plotting graphs (interpolation)



### § 2. Equation of a straight line

$y = mx + c$   
 'gradient'  $m$    'intercept'  $c$   
 $y = 3x + 1$

x	-4	-2	0	2	4
3x	-12	-6	0	6	12
+1	+1	+1	+1	+1	+1
y	-11	-5	1	7	13



### § 3. Problem solving (units – easy!)

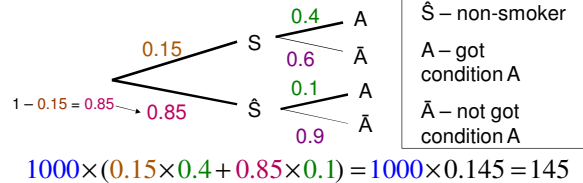
- 4 drinks cost £12
- How much do 5 drinks cost?
  - Unit is a drink
  - 1 drink costs **less than** 4 drinks so **divide** cost by 4  $\frac{£12}{4} = £3$
  - 1 drink costs  $\frac{£12}{4} = £3$
  - 5 drinks cost **more than** 1 drink so **multiply** cost by 5
  - 5 drinks cost  $£3 \times 5 = £15$

### § 3. Problem solving (units – hard!)

- It takes 24 weeks for 9 people to build 3 primary health centres (PHCs)
- How long does it take 4 people to build 6 PHCs?
  - First make PHC the unit and calculate how many weeks it takes 9 people to build 1 PHC
    - $\frac{24}{3} = 8$  weeks
  - Next make the number of people the unit and calculate how many weeks it takes 1 person to build 1 PHC
    - $8 \times 9 = 72$  weeks
  - Finally get answer by multiplying by the number of PHCs (6) and dividing by the number of people (4)
    - $\frac{72 \times 6}{4} = 108$  weeks for 4 people to build 6 PHCs

### § 3. Problem solving (probabilities)

- Suppose 15% of people are smokers and 40% of smokers get condition A while only 10% of non-smokers get condition A
- Out of 1000 people, how many would we expect to get condition A?



### § 4. Algebraic expressions (indices and roots)

$$3 \times 3 \times 3 \times 3 = 3^4$$

1      2      3      4      ← 'index' 'power' 'exponent'  
 ← 'base'

- $n \times n = n^2$  'n squared' or 'n to the power 2'
- $n \times n \times n = n^3$  'n cubed' or 'n to the power 3'
- $n \times n \times n \times n = n^4$  'n to the power 4'

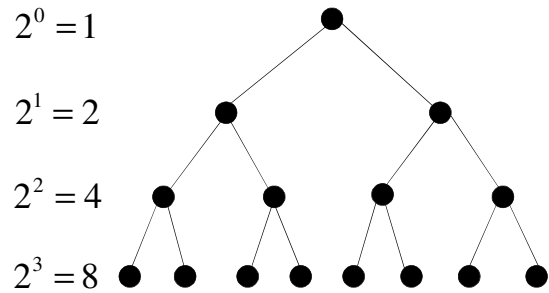
Roots can be used to undo indices:

Square root:  $\sqrt[n^2]{n^2} = n$ , (usually written as  $\sqrt{n^2} = n$ )

Cube root:  $\sqrt[3]{n^3} = n$

Fourth root:  $\sqrt[4]{n^4} = n$ , and so on

### § 4. Indices (doubling)



### § 4. Indices (rules)

$a^m \times a^n = a^{m+n}$	$4^3 \times 4^2 = (4 \times 4 \times 4) \times (4 \times 4) = 4^5 = 4^{3+2}$
$a^m \div a^n = a^{m-n}$	$\frac{4^3}{4^2} = \frac{4 \times 4 \times 4}{4 \times 4} = 4 = 4^1 = 4^{3-2}$
$(a^m)^n = a^{m \times n}$	$(4^3)^2 = (4 \times 4 \times 4)^2 = (4 \times 4 \times 4) \times (4 \times 4 \times 4) = 4^6 = 4^{3 \times 2}$
$a^{-m} = \frac{1}{a^m}$	$4^{-3} = \frac{1}{4^3}$

### § 4. Indices (more rules!)

$a^0 = 1$ (assuming $a \neq 0$ )	$4^0 = 1$
$a^{1/n} = \sqrt[n]{a}$	$4^{1/2} = \sqrt{4}$
$a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$	$4^{3/2} = \sqrt{4^3} = \sqrt{64} = 8$ and $4^{3/2} = (\sqrt{4})^3 = (2)^3 = 8$
$(a \times b)^n = a^n \times b^n$	$(4 \times 8)^2 = (4 \times 8) \times (4 \times 8) = 4 \times 4 \times 8 \times 8 = 4^2 \times 8^2$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{4}{8}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ and $\frac{4^2}{8^2} = \frac{16}{64} = \frac{1}{4}$

## § 4. Roots (just two more!)

(assuming  $a \geq 0$  and  $b \geq 0$ )

$$\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

$$\sqrt[3]{27 \times 64} = \sqrt[3]{1728} = 12 \text{ and}$$

$$\sqrt[3]{27} \times \sqrt[3]{64} = 3 \times 4 = 12$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \text{ (assuming } b \neq 0)$$

$$\sqrt{\frac{81}{9}} = \sqrt{9} = 3 \text{ and}$$

$$\frac{\sqrt{81}}{\sqrt{9}} = \frac{9}{3} = 3$$

These rules are used in the Basic Statistics module

## § 4. Indices (standard form)

$$4,000,000,000 = 4 \times 10^9$$

$$23,950 = 2.395 \times 10^4$$

$$0.00648 = 6.48 \times 10^{-3}$$

$$\begin{aligned} 4 \times 10^5 - 5 \times 10^3 &= (4 \times 100,000) - (5 \times 1,000) \\ &= 400,000 - 5,000 = 395,000 \end{aligned}$$

$$\frac{4 \times 10^7}{2 \times 10^3} = \frac{4}{2} \times 10^{7-3} = 2 \times 10^4$$

## § 5. Topics in Term 1 modules using basic maths skills

### Graphs

- Descriptive statistics (visual representation of relationship between variables)
- Linear regression

### Problem solving

- Applying basic maths skills
- Thinking through appropriate strategies using these skills

### Powers and square root

- Variance
- Standard deviation
- Standard error

### Standard form

- Calculator readout

## Intended learning objectives (achieved?)

- You should be able to:
  - understand the terminology of graphs and use axes, scales and co-ordinates
  - plot simple graphs
  - understand the equation of a straight line and use it to plot straight line graphs
  - understand and solve problems involving unit quantities
  - understand and solve problems using probability trees
  - use the rules for indices (multiply and divide powers, raise a power to a power, reciprocals)
  - understand what is meant by standard form and convert numbers to standard form

## Key rules of powers

- To multiply (quantities to) powers **OF THE SAME BASE** add the indices
- To divide (quantities to) powers **OF THE SAME BASE** subtract the indices
- To raise a power of a quantity to a power, multiply the indices
- A negative index gives the reciprocal of the quantity

N.B. For next session: <http://www.lshtm.ac.uk/edu/studyskills.html> (subheading 'Maths and Numeracy Skills')



**London School of Hygiene & Tropical Medicine  
BASIC MATHS SUPPORT SESSIONS**

**Session 3 – Graphs, Problem Solving, and Powers Notes**

**§ 1. Plotting Graphs**

**Axes**

To plot a graph we first need to draw **axes**.



The horizontal axis is the **x-axis** and the vertical axis is the **y-axis**. The intersection of these axes is the **origin**. Axes should always be labelled.

**Scale**

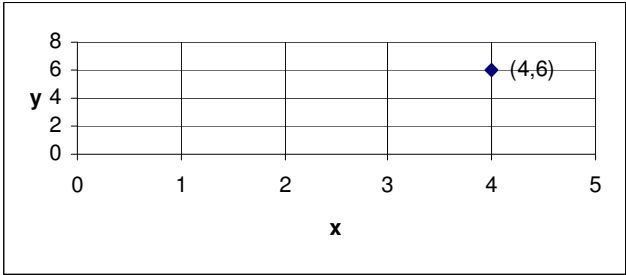
The number of units represented by a unit length along an axis is called the **scale**.

e.g. 1 cm = 2 units

We can have different scales on each axis.

**Coordinates**

These mark the points on the graph. For example the point plotted as shown has the value  $x = 4$  and  $y = 6$  and is said to have coordinates (4,6). The  $x$  value is always given first.

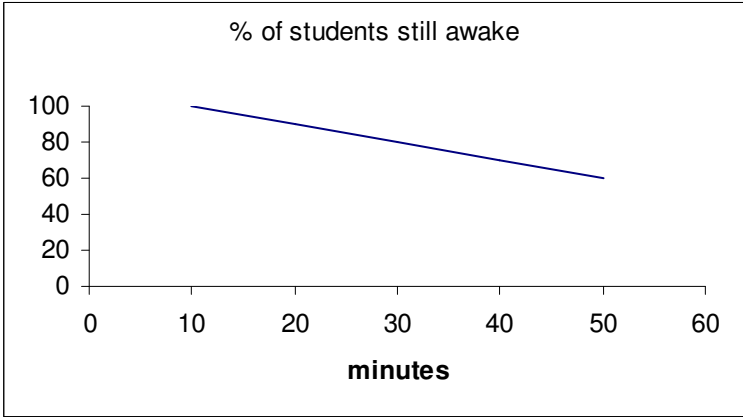


**Plotting graphs**

Every graph shows a relationship between two sets of numbers.

e.g. draw the graph of the following information:

minutes into maths session	10	20	30	40	50
% of students still awake	100	90	80	70	60



**Interpolation**

If we wish to find the number of students who are still awake after, say, 25 minutes then we find 25 on the x-axis, draw a vertical line to the graph and then draw a horizontal line to the y-axis and read off the value. If we wish to find the time after which 65% of the students are still awake then we find 65 on the y-axis, draw a horizontal line to the graph and then draw a vertical line to the x-axis and read off the value. The values here would be 85% of the students and 45 minutes respectively. What we are doing is using the graph to find approximate values for situations within the range of figures we have but for which we don't have actual data – this is **interpolation**.

**§ 2. Equation of a straight line**

Draw a graph of  $y = 2x + 3$  for values of  $x$  between  $-3$  and  $3$ .

x	-3	-2	-1	0	1	2	3
2x	-6	-4	-2	0	2	4	6
+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3
y	-3	-1	1	3	5	7	9

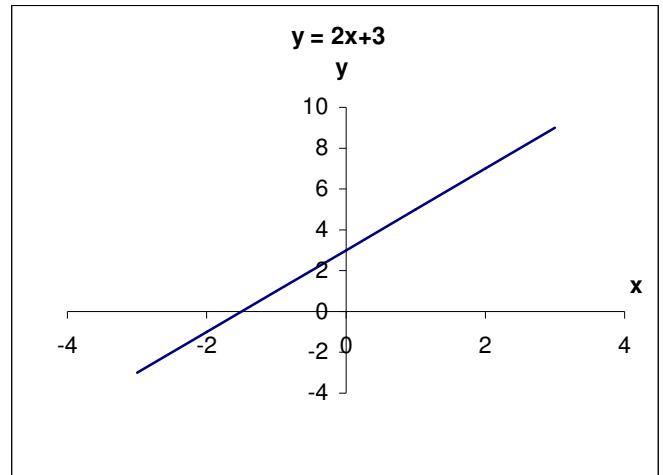
Equations of this form are called linear equations and always produce a straight-line graph. The highest powers of  $x$  and  $y$  are 1.

We really only need 2 points to draw a straight line graph, but adding a third point will act as a check for the other two points.

The equation of a straight line can be written in the following form:

$$y = mx + c$$

where **m** is the gradient (i.e. how steep the line is) and **c** is the intercept (i.e. the point at which the line crosses the  $y$  axis).

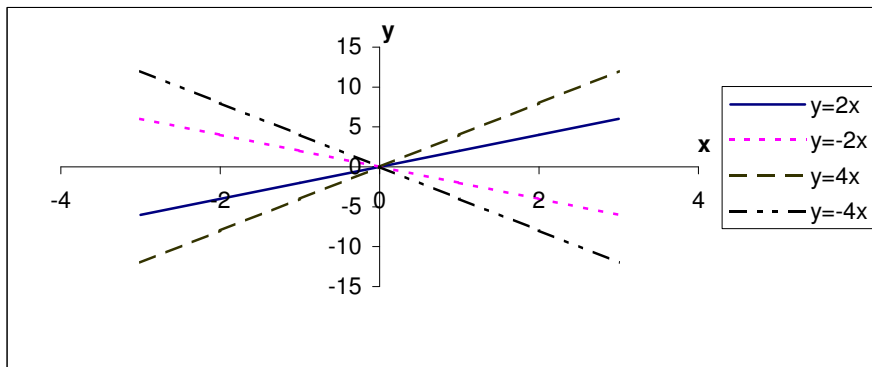


In the above example, the gradient is 2 and the intercept is 3, i.e.  $y=3$  when  $x=0$ .

Any linear equation can be written in the standard straight-line form:

e.g.  $2y + 3x = 5$  can be rewritten as  $y = -3/2 x + 5/2$  with a gradient of  $-3/2$  and intercept  $5/2$ .

The gradient can therefore be positive or negative. This indicates the direction of the slope.



In the above example the intercept for all these lines is zero. A gradient of 4 is steeper than a gradient of 2, a gradient of  $-4$  is steeper than  $-2$ . A gradient of 2 means that for every unit increase in  $x$  (increase of 1) there is double the increase in  $y$ . A gradient of  $-2$  means that for every unit increase in  $x$  (increase of 1) there is double the **decrease** in  $y$ . Note that the line slopes in the opposite direction.

### § 3. Problem Solving

We consider two general types of problem: problems involving ‘unit’ quantities, and problems involving ‘probabilities’. Accurate problem solving requires us to express the problem in the correct terms. For example, if we express a problem in terms of the wrong fractions, then however good we are at multiplying fractions, we will still get the wrong answer!

#### **1) Problems involving calculation of ‘unit’ quantities (e.g. cost, time etc..)**

There are many different ways of solving this type of problem. The method outlined here is perhaps the easiest to understand – even if it not necessarily the quickest. An important stage in this method is the calculation of the ‘unit’ quantity:

#### **Example 1**

*3 pens cost £6. How much do 4 pens cost?*

Strategy:

i) Identify the item we need to treat as a unit in order to solve the problem, it could be: a) the cost of one pen or b) the number of pens you can buy for £1. Here we need to find the cost of one pen since we can find the cost of 4 pens once we know the cost of 1 pen. Our strategy will be first to solve:

- 1 pen costs ?

ii) To find the missing ?, first identify whether 1 pen will cost more or less than 3 pens. If **less**, we divide **£6** by **3** to get the answer:

- 1 pen costs  $£6/3 = £2$ .

iii) Next, identify whether 4 pens will cost more or less than 1 pen. If **more**, we **multiply** the unit cost by 4:

- 4 pens cost  $4 \times \text{unit cost} = 4 \times £2 = £8$ .

**Important principle: if the numbers involved are horrible, investigate what the strategy would be with very simple numbers, and exactly the same pattern will work for the horrible numbers:**

#### **Example 2**

*41 pens cost £18.34; How much do 34 pens cost?*

- 1 pen costs  $£18.34/41 = £0.45$
- 34 pens cost  $34 \times £0.45 = £15.30$

We can see that the pattern corresponds exactly to that in Example 1.

#### **Example 3**

*It takes 6 doctors 3 hours to see all patients in a clinic. How long will it take 4 doctors to see all patients in a clinic?*

- 1 doctor will take (longer: 6 times longer than 6 doctors)  $6 \times 3 = 18$  hours.
- 4 doctors will take (shorter, the time for 1 doctor divided by 4)  $18/4 = 4.5$  hours.

**Example 4**

*It takes 5 hours for 4 people to paint 2 rooms; how long will it take 3 people to paint 4 rooms?*

Strategy:

This is best done in stages: work on one variable at a time, keeping the other fixed. First we'll keep the number of people the same as in the first statement, 4, and make rooms the unit:

- 4 people paint 1 room in ? (less time than 2 rooms) so  $5/2 = 2.5$  hours
- Next, find the time needed to paint 4 rooms, still keeping the number of people fixed:
- 4 people paint 4 rooms in (more time than 1 room):  $4 \times 2.5 = 10$  hours.
- Next find the time needed for one person to paint 4 rooms:
- 1 person paints 4 rooms in ( $4 \times$  more time than 4 people):  $4 \times 10 = 40$  hours
- Finally:
- 3 people paint 4 rooms in (less than one person so divide by 3)  $= 40/3 = 13.3$  hours.

**Example 5**

*In a group of 40 people with heart disease observed over 5 years, 2 suffered a heart attack. If 120 similar people were observed over 10 years, how many heart attacks would be expected?*

With more confidence, we can make short cuts instead of going down to the 'unit' each time:

- 120 people followed for 5 years will have proportionally more heart attacks than 40 people: they will have  $120/40 = 3$  times more heart attacks or  $3 \times 2 = 6$  heart attacks in total.
- 120 people followed for 10 years would be expected to suffer double the number of heart attacks that were observed over 5 years:  
i.e.  $10/5 \times 6 = 2 \times 6 = 12$  heart attacks.

We get the same answer if we follow the 'unit' method:

- 1 person followed for 5 years would be expected to have  $2/40 = 1/20 = 0.05$  heart attacks. (Although this seems like nonsense, it is extremely useful nonsense).
- 120 people observed for 5 years would be expected to have  $120 \times 0.05 = 6$  heart attacks
- 120 people observed for 10 years would be expected to have ( $10/5$  more):  $2 \times 6 = 12$  heart attacks.

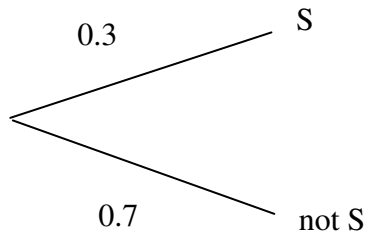
**2) Problems involving 'probabilities'**

'Probabilities' can be regarded as a type of proportion: if 20% of births are caesarean, we can say that the probability of a randomly chosen individual having been born through a caesarean birth is 20% or 1 in 5. When solving probability problems it is often necessary to work out how the probabilities of 'events' are combined to reach a particular outcome or combination of outcomes. We will look at using 'tree' diagrams as a strategy for solving these problems.

**Example 1**

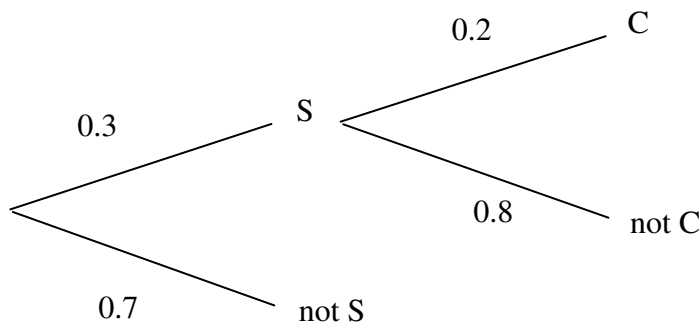
20% of smokers develop a certain medical condition, *C*, whereas only 10% of non-smokers develop condition *C*. If 30% of people are smokers, how many will develop condition *C* out of a sample of 100 people chosen at random?

i) First branch of the tree:



- First we divide the people into whether or not they are smokers: we don't need to specify that there are 100 yet, as we will concentrate just on the proportions until we're ready.
- We've translated the percentages into decimal fractions.
- Notice that we've assumed that 30% are smokers as stated above.
- Although we haven't been told what proportion are non-smokers, we can work it out by assuming it's the rest: 70%.

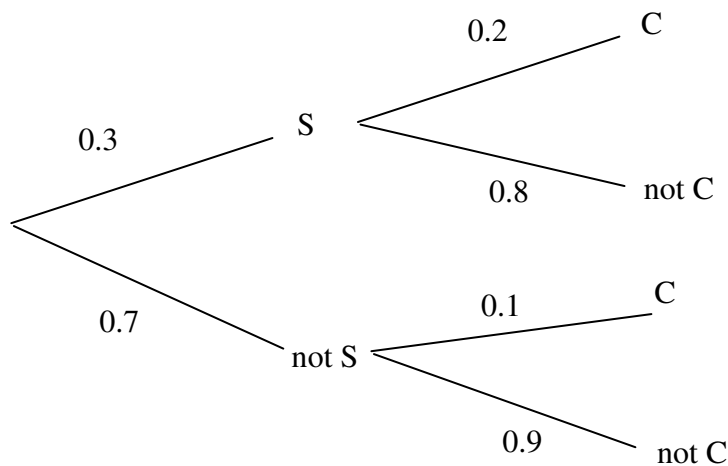
ii) The next stage is to consider what happens at each tip of the tree: first consider what happens if you're a smoker at S:



- 20% of smokers develop *C*, so 80% of smokers don't
- The 20% of smokers with condition *C* are 20% of the 30% of smokers from the first branch.
- 20% of 30% is  $0.2 \times 0.3 = 0.06$  or 6%.

- We have worked out so far that 6% of people are smokers who develop *C*, by multiplying  $0.2 \times 0.3$ . We can check this: imagine starting with 100 people: 30 will follow the smokers branch. Of the 30 ending up at S, 20% will follow the *C* branch; which is  $1/5$  of 30:  $1/5 \times 30 = 6$  smokers who develop *C*.

iii) Now we can consider the not S branch:



- 10% of non-smokers develop C, so 90% don't.
- Since 70% of people are non-smokers and 10% of these develop C, 10% of 70% develop C along this branch, which is  $0.1 \times 0.7 = 0.07$  or 7%

- We've now worked out that 7% of people are non-smokers who develop C by multiplying  $0.1 \times 0.7$ . We can check this: suppose we start with 100 people, 70 will follow the non-smokers branch. Of the 70 ending up at not S, 10% will follow the C branch; which is  $1/10$  of 70:  $1/10 \times 70 = 7$  non-smokers who develop C.

**Answer:** out of 100 people, 6 smokers will develop C and 7 non-smokers will develop C, making **13** in total.

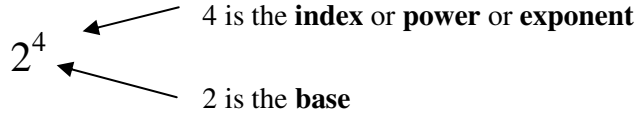
Finally, we can translate our tree into a 2 x 2 table dividing up the 100 people into 4 categories, each corresponding to a *final tip* of the tree.

We should ensure that we can match these 4 shaded cells with the 4 tips, and that we can complete the values for all the totals:

	develop C	don't develop C	totals
smokers	<b>6</b> [ $0.2 \times 0.3 \times 100$ ]	<b>24</b> [ $0.8 \times 0.3 \times 100$ ]	<b>30</b>
non-smokers	<b>7</b> [ $0.1 \times 0.7 \times 100$ ]	<b>63</b> [ $0.9 \times 0.7 \times 100$ ]	<b>70</b>
totals	<b>13</b>	<b>87</b>	<b>100</b>

**§4. Indices (powers) and Roots**

We can write  $2 \times 2$  as  $2^2$  (2 squared or 2 to the power 2)  
 $2 \times 2 \times 2$  as  $2^3$  (2 cubed or 2 to the power 3)  
 $2 \times 2 \times 2 \times 2$  as  $2^4$  (2 to the power 4) etc.

This is called index notation:  $2^4$  

The **index** tells us how many times the **base** is multiplied by itself and is a shorthand way of presenting the information.

We can also use letters instead of numbers and write:

$m \times m$  as  $m^2$  (m squared or m to the power 2)  
 $m \times m \times m$  as  $m^3$  (m cubed or m to the power 3)  
 $m \times m \times m \times m$  as  $m^4$  (m to the power 4)  
 etc.

**Rules for indices**

1) To **multiply powers** of the same base **add the indices**.

$$a^m \times a^n = a^{m+n}$$

e.g.  $2^3 \times 2^4 = 2^{3+4} = 2^7 = 128$   
 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7 = 128$

2) To **divide powers** of the same base **subtract the indices**.

$$a^m \div a^n = a^{m-n}$$

e.g.  $2^5 \div 2^3 = 2^{5-3} = 2^2 = 4$   
 $\frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} = \frac{2 \times 2}{1} = 4$

3) To **raise a power of a quantity to a power, multiply the indices**.

$$(a^m)^n = a^{mn}$$

e.g.  $(2^2)^3 = 2^{2 \times 3} = 2^6 = 64$   
 $2^2 \times 2^2 \times 2^2 = 2^6 = 64$

4) A **negative index** is the **reciprocal of the quantity**.

$$a^{-m} = \frac{1}{a^m}$$

e.g.  $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

Why does this work?  $2^2 \div 2^5 = 2^{-3}$  using the division rule and we get a negative power.

It also equals  $\frac{2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2 \times 2 \times 2} = \frac{1}{2^3}$

5) Any quantity raised **to the power zero is 1**.

$$a^0 = 1$$
 (provided a is not zero)  
 e.g.  $2^0 = 1$

**The rules for indices apply to positive, negative and fractional indices.**

We write  $2 \times 2 = 2^2 = 4$  i.e. 2 squared equals 4. Conversely, the **square root** of 4 equals 2.  
 Similarly,  $2 \times 2 \times 2 = 2^3 = 8$  i.e. 2 cubed equals 8 and conversely the **cube root** of 8 equals 2.  
 Also  $2 \times 2 \times 2 \times 2 = 2^4 = 16$  i.e. 2 to the power 4 equals 16 and so the **fourth root** of 16 equals 2.

In index notation, a power that is a fraction of the form  $1/n$  represents the  $n^{\text{th}}$  root of a number.

$$a^{1/n} = \sqrt[n]{a} = n^{\text{th}} \text{ root of } a$$

$$a^{1/n} \times a^{1/n} \times \dots \times a^{1/n} = a^{n/n} = a^1 = a$$

e.g.  $4^{1/2} = \sqrt{4} = 2,$        $8^{1/3} = \sqrt[3]{8} = 2,$        $16^{1/4} = \sqrt[4]{16} = 2$

$2 \times 2 = 4$   
 $\sqrt{4} \times \sqrt{4} = 4$   
 $4^{1/2} \times 4^{1/2} = 4$

In index notation, a power that is a fraction of the form  $m/n$  represents the  $m^{\text{th}}$  power of the  $n^{\text{th}}$  root of a number.

$$a^{m/n} = \sqrt[n]{a^m} = (n^{\text{th}} \sqrt{a})^m$$

$$a^{m/n} \times a^{m/n} \times \dots \times a^{m/n} = a^{n \times m/n} = a^m$$

e.g.  $4^{3/2} = \sqrt{4^3} = \sqrt{64} = 8$  or  $4^{3/2} = (\sqrt{4})^3 = (2)^3 = 8$

▶▶ On a calculator, look for the following keys  $\sqrt{\quad}, x^2, \sqrt[3]{\quad}, x^3, \sqrt[x]{\quad}, x^\square$  or  $\wedge$  or  $x^y$  which are associated with finding powers and roots. On **CASIO fx-85GT PLUS**,  $\sqrt{\square}$  is the square root key (use **SHIFT** and this key for cube root), and to raise a number to a power you can use the  $x^\square$  key (use **SHIFT** and this key for roots). For example, for  $4^3$  key in:  $4 x^\square 3 =$  to get 64, or for  $\sqrt[4]{81}$  key in:  $4 \text{ SHIFT } x^\square 81 =$  to get 3.

**Rules for roots**

1) The  $n^{\text{th}}$  root of a product is the product of the  $n^{\text{th}}$  roots of the numbers.

$$\sqrt[n]{(ab)} = \sqrt[n]{a} \sqrt[n]{b}$$

e.g.  $\sqrt[3]{27 \times 64} = \sqrt[3]{1728} = 12$   
 $\sqrt[3]{27} \times \sqrt[3]{64} = 3 \times 4 = 12$

2) The  $n^{\text{th}}$  root of a quotient is the quotient of the  $n^{\text{th}}$  roots of the numbers.

$$\sqrt[n]{(a/b)} = \sqrt[n]{a} / \sqrt[n]{b}$$

(Quotient is a term which means the result of a division).

e.g.  $\sqrt{81/9} = \sqrt{9} = 3$   
 $\sqrt{81} / \sqrt{9} = 9/3 = 3$

**Standard form**

We can write any number as a value between 1 and 10 multiplied by a power of 10. Again, this is useful as a shorthand notation for very large or very small numbers.

e.g.  $3\,000\,000 = 3 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 3 \times 10^6$

**To change a whole number to standard form**, put a decimal point after the first digit and count how many digits are left – this gives the power of 10.

e.g.  $1\,385 = 1.385 \times 10^3$   
 $1\,385\,000\,000\,000 = 1.385 \times 10^{12}$

**To change a decimal to standard form**, put a decimal point after the first non zero digit and count how many digits there are from the original position of the point to the new position – this gives the negative power of 10.

e.g.  $0.000789 = 7.89 \times 10^{-4}$



**To add or subtract numbers in standard form**, convert back to ordinary numbers to do the calculation.

e.g. Add  $3 \times 10^4$  to  $2 \times 10^6$

$$3 \times 10^4 + 2 \times 10^6 = (3 \times 10,000) + (2 \times 1,000,000) = 30,000 + 2,000,000 = 2,030,000 = 2.03 \times 10^6$$

**To multiply or divide numbers in standard form**, use the ordinary rules of indices.

e.g. Multiply  $3 \times 10^4$  by  $5 \times 10^6$

$$3 \times 10^4 \times 5 \times 10^6 = (3 \times 5) \times (10^4 \times 10^6) = 15 \times 10^{10} = 1.5 \times 10^{11}$$

▶▶ On **CASIO fx-85GT PLUS** calculator, the result of a calculation will automatically be displayed in exponential display format if the number has too many digits to fit on the display. To enter a number in standard form: e.g. for  $1.9 \times 10^{-3}$ , key in: 1.9 [**x10<sup>x</sup>**] - 3  
(Other calculators, e.g. older models, may have an **EXP** key to use instead of the [**x10<sup>x</sup>**] key.)

**London School of Hygiene & Tropical Medicine  
BASIC MATHS SUPPORT SESSIONS**

**Session 3 – Graphs, Powers and Problem Solving Exercises**

You may prefer to use a calculator for some of these exercises.

If you need help with using your calculator please consult your calculator manual and see the information on calculator use offered in Basic Statistics for PHP or Statistics for EPH (STEPH) or Statistics with Computing.

**§ 1. Plotting Graphs**

1) The values below give corresponding values of x and y. Using a scale of 1 cm to 2 units on the x-axis and 1 cm to 5 units on the y-axis draw the graph of these points.

x	0	2	4	6	8	10
y	2	8	14	20	26	32

**§ 2. Equation of a straight line**

- 1) i) Draw a graph of  $y = 5x - 2$  for values of x between  $-3$  and  $3$ .
  - ii) On the same set of axes draw the graph of  $y = 2 - 3x$  between the same values of x. At what point do the two lines intersect (cross)?
  - iii) What are the gradients and intercepts of the 2 lines?
  - iv) For both lines, what are the values of x when  $y = 0$  (i.e. at what x value does each line cross the x-axis)?

**§ 3. Problem Solving**

- 1) A health centre sees on average 52 patients every 3 weeks.
 

How many will it see every 4 weeks on average?
- 2) 55g of a drug is enough to treat 45 people.
  - i) How many people will 40g treat?
  - ii) How much of the drug will be required by 22 people?
- 3) Out of 450 asthmatic patients followed up for 6 years, 64 had an asthma-related hospital visit.
  - i) How many such visits would you expect if 1 person was followed up for 1 year? [This might seem a silly calculation but it's actually a very useful quantity, since it allows both further calculation and comparisons. It's known as the *rate per person-year*.]
  - ii) How many visits would you expect if 12 people were followed up for 3 years?
  - iii) Another group of 390 asthmatics were observed for 5 years, and out of these 59 had an asthma-related hospital visit. Is this second group more or less ill than the first, in terms of hospital visits?

4) An ambulance travels from hospital A to hospital B in 30 minutes when its average speed is 35km/hour.

How long would it take for this journey travelling at 55km/hour?

5) In a maternity unit on average 20% of births are caesarean; suppose that a certain complication tends to occur on average in 10% of caesarean births, but only in 5% of otherwise normal births. Out of 100 births in this maternity unit, how many would you expect:

i) to have this complication following a caesarean?

ii) to have this complication regardless of the type of birth?

6) Suppose that proportion  $p$  of the adult population have a gene that makes them vulnerable to condition D. People with this gene have a chance  $q$  of developing the condition, whereas people without the gene have only chance  $r$  of developing D.

Out of  $N$  people taken at random from the adult population, how many would you expect to develop the condition in terms of  $N$ ,  $p$ ,  $q$  and  $r$ ; (hint: the proportion without the gene =  $1-p$ )?

**§ 4. Indices (powers) and Roots**

1) Write the following using indices:

a)  $3 \times 3 \times 3 \times 3 \times 3$

b)  $\sqrt{t \times t \times t}$

2) Simplify the following:

a)  $a^3 \times a^5$

b)  $(a^3)^5$

c)  $\frac{a^4 \times a^6}{a^3 \times a^5}$

3) Find the values of the following:

a)  $2^3 \times 3^2$

b)  $6^{-2}$

c)  $8^0$

d)  $27^{1/3}$

e)  $27^{-1/3}$

4) Use the rules for roots to find the value of  $\sqrt{2^6 \times 5^4}$

5) Put the following into standard form:

a) 723

b) 72300

c) 0.0723

6) Work out the following and give the answer in standard form:

a)  $(4 \times 10^3) + (3 \times 10^2)$

b)  $(4 \times 10^3) \times (3 \times 10^2)$

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BASIC MATHS SUPPORT SESSIONS**

**Session 3 – Graphs, Powers and Problem Solving – Further Exercises**

These are intended to supplement the exercises you have already done and extend the concepts.

You **will** need a calculator for some of these exercises. If you need help with using your calculator please consult your calculator manual and see the information on calculator use offered in Basic Statistics for PHP or Statistics for EPH (STEPH) or Statistics with Computing.

**§ 1. and 2. Graphs**

1) The values below give corresponding values of % of students attending lectures and weeks into the term. Using a suitable scale, draw the graph of these points.

Weeks into term	4	8	12
% of students in lectures	85	70	55

- Using the graph, find the % of students attending lectures after i) 3 weeks ii) 11 weeks
- Using the graph, find out approximately how many weeks into term we expect 63% student attendance.

**§ 3. Problem Solving**

1) If it takes 3 hours for 5 people to enter 250 questionnaires into a computer, how long will it take 4 people to enter 300 questionnaires?

2) In a sample of 110 people there are 40 men and 70 women. If 52 of these people smoke, how many male smokers would you expect there to be if there was no difference in the smoking habits of men and women.

3) If the level of a metabolite in the urine is reduced by 0.03g/l for every additional 0.15g dosage of a drug, what amount of metabolite would you expect to be excreted by someone with pre-treatment level of 1.6g/l after being given a dose of 2.2g drug?

4) Suppose 10% of the adult population are ex-smokers, 20% are current smokers, and the rest are 'never' smokers. Also suppose that the chance of developing condition C is 0.3, 0.5 and 0.1 among ex-smokers, current smokers and 'never' smokers respectively. Out of 1000 adults taken at random, how many will you expect to get condition C? [Hint: if you use a tree, it will have 3 branches to start off with...]

5) Suppose (as in one of the session exercises) that proportion  $p$  of the adult population have a gene that makes them vulnerable to condition D. People with this gene have a chance  $q$  of developing the condition, whereas people without the gene have only chance  $r$  of developing D. Now, among those who have the gene and go on to develop the condition, 60% die from the condition; whereas only 20% die from the condition among people without the gene who develop D. Question: (i) Out of  $N$  people taken at random from the adult population, how many would you expect (in terms of  $N$ ,  $p$ ,  $q$  and  $r$ ) to die from this condition? (ii) If  $N=1,000$ , how many would you expect to die from this condition? [Hint: The tree will have a further set of branches compared to the tree in the session exercises.]

**§ 4. Indices (powers) and Roots**

1) Without using a calculator, find the value of :

a)  $\frac{2^4 \times 3^2}{4^2}$       b)  $(6^{-2})^{-1}$       c)  $87.5^0$       d)  $27^{2/3}$       e)  $(27/8)^{-2/3}$       f)  $\frac{5^2 \times 4^2}{2^2}$

Note that  $(a \times b)^n = a^n \times b^n$  and  $(a / b)^n = a^n / b^n$

2) Find the value of n in each of the following:

i)  $12^4 \times 12^n = 12^9$       ii)  $3^4 \div 3^n = 3^6$       iii)  $1/8 = 2^n$

3) Calculate the value of  $\frac{z \div z^{-1/2}}{z^{1/3}}$  when  $z = 64$ .

4) Using the rules for roots, find the value of :  $\frac{\sqrt{2^9 \times 3^5}}{\sqrt{2 \times 3}}$

5) Using a calculator find the answers to the following:

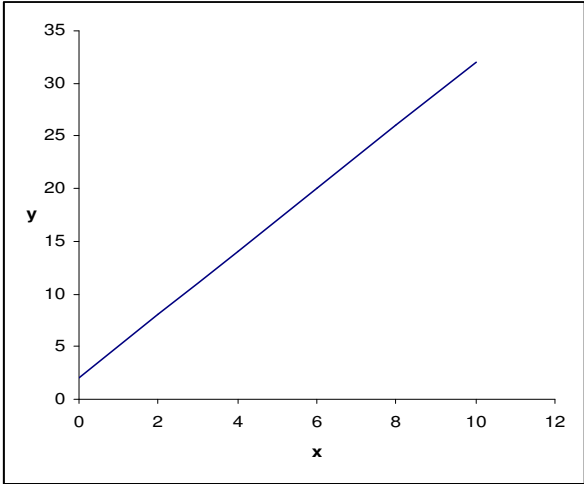
a)  $3.764 \times 10^2 + 2.98 \times 10^3$       b)  $3.764 \times 10^{-2} - 2.98 \times 10^{-4}$       c)  $3.764 \times 10^{-2} \times 2.98 \times 10^4$

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**Session 3 – Graphs, Powers and Problem Solving - Solutions**

**§ 1. Plotting Graphs**

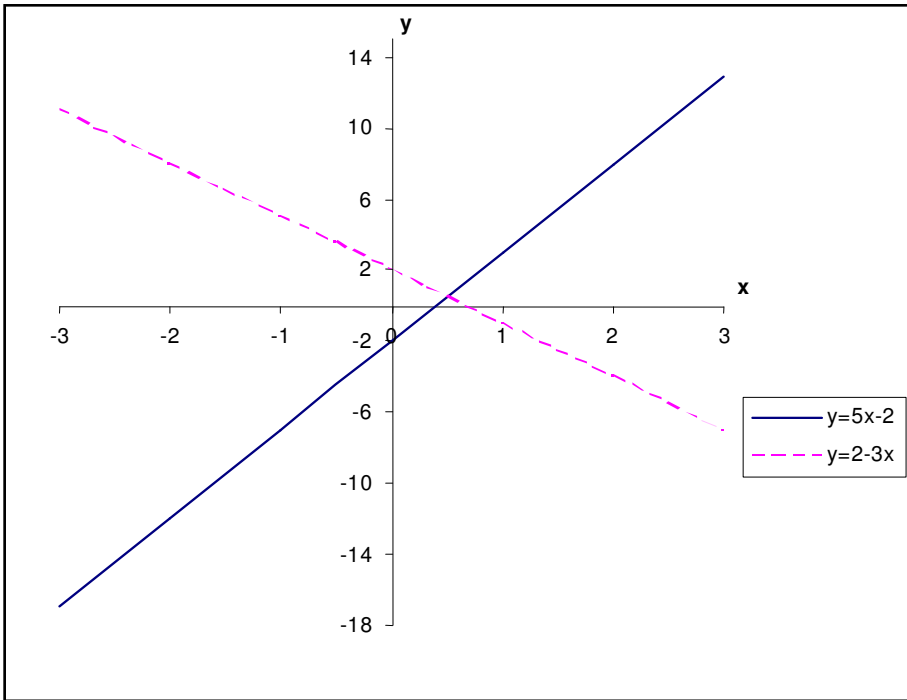
1)



**§ 2. Equation of a straight line**

1) i)

x	-3	-2	-1	0	1	2	3
$y = 5x - 2$	-17	-12	-7	-2	3	8	13
$y = 2 - 3x$	11	8	5	2	-1	-4	-7



- ii) The lines cross at the point  $(1/2, 1/2)$ .
- iii) Gradient and intercept of the line  $y = 5x - 2$  are gradient = 5 and intercept = -2.  
Gradient and intercept of the line  $y = 2 - 3x$  are gradient = -3 and intercept = 2.

- iv) The line  $y=5x-2$  crosses the x-axis when  $y=0$  so we can read off the value  $x=2/5$  from the graph. We can also calculate the value so  $5x-2 = 0$  therefore  $5x=2$  and  $x=2/5$ .  
The line  $y=2-3x$  crosses the x-axis when  $y=0$  so we can read off the value  $x=2/3$  from the graph. We can also calculate the value so  $2-3x = 0$ , therefore  $3x=2$  and  $x=2/3$ .

### § 3. Problem Solving

- 1) In 1 week will see (less)  $52/3 = 17.3$ ; in 4 weeks will see  $4 \times 17.3 = 69.2$   
Number of patients seen is 69.
- 2) i) 1g will treat  $45/55 = 0.82$  people; 40g will treat  $40 \times 0.82 = 32.8$   
Can only treat 32 people (even though mathematically 32.8 rounds up to 33).  
ii) 1 person requires  $55/45 = 1.22$ g; 22 people will require  $22 \times 1.22 = 26.9$ g.
- 3) i) 450 people followed up for 1 year would have  $64/6 = 10.67$  visits; 1 person followed up for 1 year would have  $10.67/450 = 0.024$  visits. This is the number of visits per person-year.  
ii) Two ways of seeing this:  
a) 12 people followed up for 1 year would be 12 times the 1 person rate =  $12 \times 0.024 = 0.29$ ; so for 3 years:  $3 \times 0.29 = 0.86$ .  
b) 12 people followed up for 3 years = 36 person years.  
So the answer will be  $36 \times$  the rate for 1 person-year =  $36 \times 0.024 = 0.86$ .  
iii) 390 people followed-up for 1 year would have  $59/5 = 11.8$  visits; so 1 person would have  $11.8/390 = 0.030$  visits; this rate per person-year is slightly higher than that in the first group, so they are slightly 'illier' in this respect.
- 4) If it travelled at 1km/hour it would take longer:  $35 \times 30$  minutes = 1050 minutes. [Notice that the fact that the unit of time, minutes, is different from that involved in the speed, hours, is irrelevant: what's important is simply how much slower it is...] So if it travelled at 55km/hour it would take less:  $1050/55 = 19.1$  minutes.
- 5) i) The tree should go: Caesarean branch: 0.2 Caesarean >> 0.1 complication and 0.9 no complication. So  $0.1 \times 0.2 \times 100$  are Caesarean with complication = 2.  
ii) Normal birth branch: 0.8 normal birth >> 0.05 complication and 0.95 no complication.  
So  $0.05 \times 0.8 \times 100$  are normal births with complication = 4.  
So total complications are  $2 + 4 = 6$ .
- 6) For those developing the gene, the gene branch goes  $p$  have gene >>  $q$  develop D. So out of  $N$  people,  $Npq$  will develop D through this branch. For those not developing the gene, this branch will go  $(1-p)$  no gene >>  $r$  develop D. So out of  $N$  people,  $N(1-p)r$  will develop D through this branch. So the total number developing the condition =  $Npq + N(1-p)r$  or  $N(pq + (1-p)r)$ .

### § 4. Indices (powers) and Roots

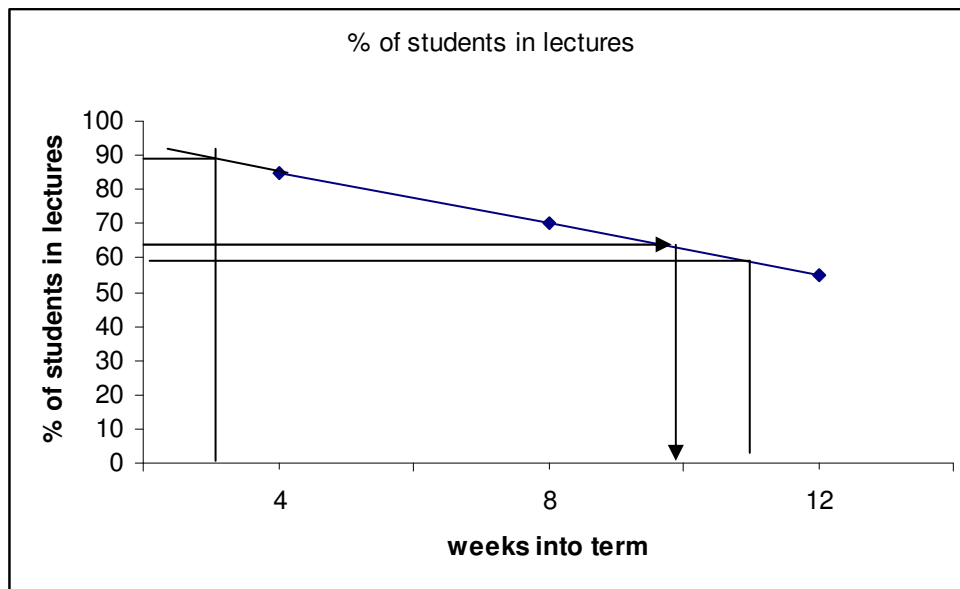
- 1) a)  $3^5$       b)  $(t^3)^{1/2} = t^{3/2}$
- 2) a)  $a^{3+5} = a^8$       b)  $a^{3 \times 5} = a^{15}$       c)  $\frac{a^{10}}{a^8} = a^{10-8} = a^2$
- 3) a)  $2^3 \times 3^2 = 8 \times 9 = 72$       b)  $6^{-2} = 1/6^2 = 1/36$       c) 1  
d)  $27^{1/3} = \sqrt[3]{27} = 3$       e)  $27^{-1/3} = 1/\sqrt[3]{27} = 1/3$
- 4)  $\sqrt{2^6 \times 5^4} = \sqrt{2^6} \times \sqrt{5^4} = 2^3 \times 5^2 = 8 \times 25 = 200$
- 5) a)  $7.23 \times 10^2$       b)  $7.23 \times 10^4$       c)  $7.23 \times 10^{-2}$
- 6) a)  $4000 + 300 = 4300 = 4.3 \times 10^3$       b)  $4 \times 3 \times 10^3 \times 10^2 = 12 \times 10^5 = 1.2 \times 10^6$

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BASIC MATHS SUPPORT SESSIONS**

**Session 3 – Graphs, Powers and Problem Solving – Further Exercises  
Solutions**

**§ 1. and 2. Graphs**

1)



- a) i) After 3 weeks 88.75% of students are attending
- ii) After 11 weeks 58.75 % of students are attending
- b) After approximately 10 weeks (62.5% of students are still attending after 10 weeks)

**§ 3. Problem Solving**

1) To enter 250 questionnaires it takes 1 person more time:  $5 \times 3 = 15$  hours; so it takes 1 person  $15/250 = 0.06$  hours to enter just 1 questionnaire. So to enter 300 questionnaires it will take 1 person  $300 \times 0.06 = 18$  hours; finally, it will take 4 people less:  $18/4 = 4.5$  hours.

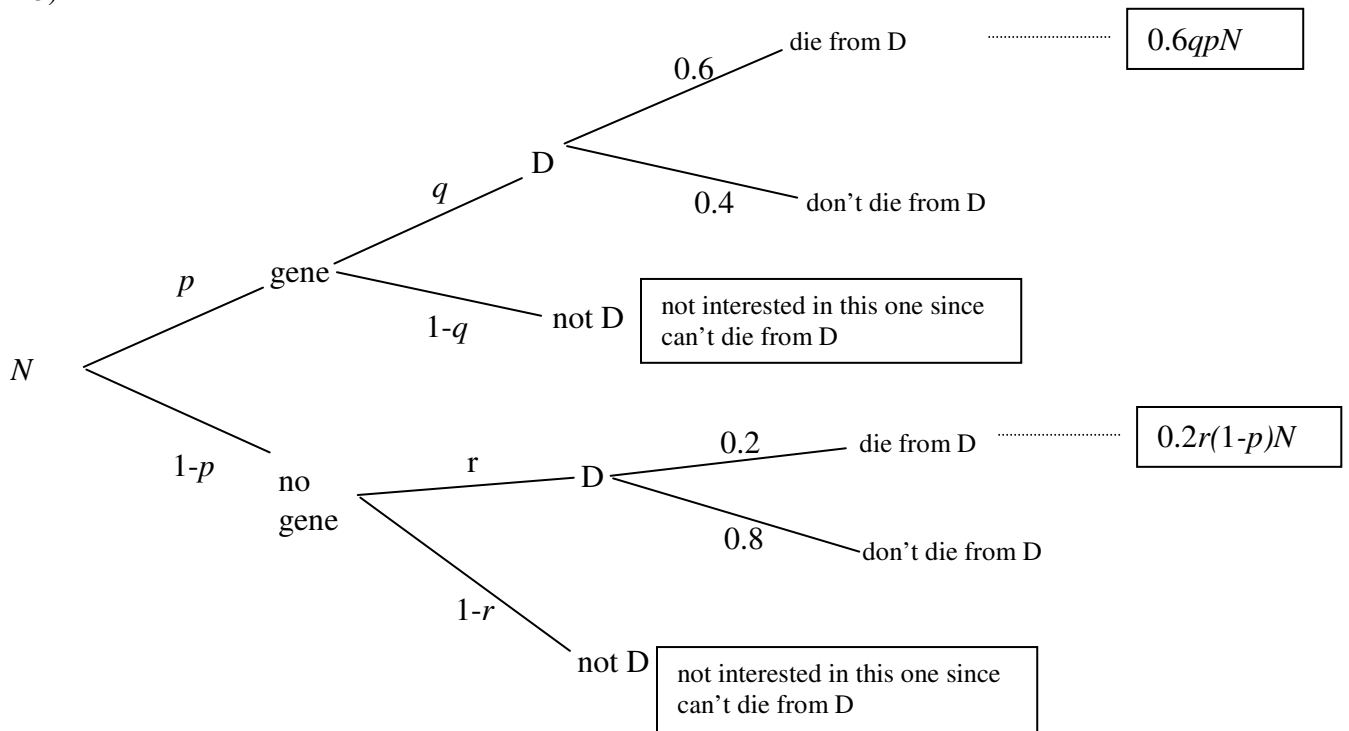
2) The proportion of smokers in the sample is  $52/110$ . We would expect exactly this proportion of men to smoke if there was no difference in the smoking habits of men and women. So we would expect  $(52/110) \times 40$  men to smoke = 19 (rounded up to the nearest person).

3) 1g of drug would reduce metabolite concentration by (more than 0.15g drug):  
 $(1/0.15) \times 0.03 = 6.67 \times 0.03 = 0.2\text{g/l}$ ;  
 so 2.2g will reduce concentration by  $2.2 \times 0.2 = 0.44\text{g/l}$ .  
 Since the pre-treatment level was 1.6g/l, we would expect post treatment level to be  $1.6 - 0.44 = 1.16\text{g/l}$ .  
 Alternatively, 2.2g drug would reduce by  $(2.2/0.15) \times 0.03 = 0.44$ , and again we would expect post treatment level to be  $1.6 - 0.44 = 1.16\text{g/l}$ .

4) We expect  $(0.3 \times 0.1 \times 1000)$  ex-smokers,  $(0.5 \times 0.2 \times 1000)$  current smokers and  $(0.1 \times 0.7 \times 1000)$  'never' smokers to develop the condition:  $30 + 100 + 70 = 200$ .



5)



(ii) Since  $N = 1000$ , total =  $600qp + 200r(1-p)$  (i) Total:  $0.6qpN + 0.2r(1-p)N$

**§ 4. Indices (powers) and Roots**

1) a)  $\frac{16 \times 9}{16} = 9$       b)  $(6^{-2})^{-1} = 6^2 = 36$       c) 1      d)  $(\sqrt[3]{27})^2 = 3^2 = 9$   
 e)  $\frac{1}{(\sqrt[3]{\frac{27}{8}})^2} = 1 \div (3/2)^2 = 1 \div 9/4 = 4/9$       f)  $(5 \times 4 / 2)^2 = (20/2)^2 = (10)^2 = 100$

2) i)  $12^n = 12^9 \div 12^4 = 12^5$  so  $n = 5$     can also say  $12^{4+n} = 12^9$  so  $4 + n = 9$  and  $n = 5$

ii)  $3^{4-n} = 3^6$  so  $4 - n = 6$  and  $n = -2$

iii)  $1/8 = 1/2^3 = 2^{-3}$  and  $n = -3$

3)  $\frac{64 \div 64^{-1/2}}{64^{1/3}} = \frac{64 \div 1/\sqrt{64}}{\sqrt[3]{64}} = \frac{64 \div 1/8}{4} = \frac{64 \times 8}{4} = 128$

4)  $\frac{\sqrt{2^9 \times 3^5}}{\sqrt{2 \times 3}} = \sqrt{\frac{2^9 \times 3^5}{2 \times 3}} = \sqrt{2^8 \times 3^4} = \sqrt{2^8} \times \sqrt{3^4} = 2^4 \times 3^2 = 16 \times 9 = 144$

5) a)  $3.764 \times 10^2 + 2.98 \times 10^3 = 376.4 + 2980 = 3356.4 = 3.3564 \times 10^3$

b)  $3.764 \times 10^{-2} - 2.98 \times 10^{-4} = 0.03764 - 0.000298 = 0.037342 = 3.7342 \times 10^{-2}$

c)  $3.764 \times 10^{-2} \times 2.98 \times 10^4 = 1121.672 = 1.121672 \times 10^3$

# Basic Maths

## Session 4: Logarithms

### Intended learning objectives

- At the end of this session you should be able to:
  - understand the concept of logarithms, inverse logs and natural logs
  - use the rules of logs
  - use the log function on the calculator
  - transform non-linear to straight line graphs using logs

### § 1. Logarithms (activity – part 1)

- Plot the following coordinate points on graph paper

x	100	1,000	10,000	100,000	1,000,000
y	0.001	0.0001	0.00001	0.000001	0.0000001

### § 1. Logarithms (basics)

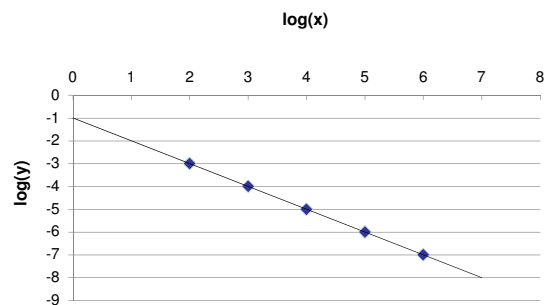
- A **log is the power** you have to raise the base to in order to get the number
- **Powers of 10** are '**logarithms**' to base 10
- $\log_{10} 1000 = 3$   
 base number power ('exponent')
- $\text{antilog}_{10} 3 = 10^3 = 1000$   
 ('antilogarithm' or 'inverse logarithm')

### § 1. Logarithms (activity – part 2)

- Use log button on calculator to convert (x,y) coordinates and plot (log(x), log(y)) on graph paper

x	100	1,000	10,000	100,000	1,000,000
log(x)	2	3	4	5	6
y	0.001	0.0001	0.00001	0.000001	0.0000001
log(y)	-3	-4	-5	-6	-7

### § 1. Logarithms (activity – part 2)



## § 1. Logarithms (uses)

- Logarithms make very large or very small numbers easier to handle
- Logarithms convert quite complicated mathematical manipulations into easier forms
- Logarithms can be used to convert curved graphs into straight-line graphs to determine the exact values in the relationship between variables

## § 1. Logarithms and exponentials (applications)

### Examples of uses of logarithms

- Richter scale for earthquakes uses logarithm scale
- pH scale for acidity of substances
- Exponential and logistic population growth models
- Exponential decay of drug concentration in a patient's body
- Decibel scale for the power of sound uses logarithm scale

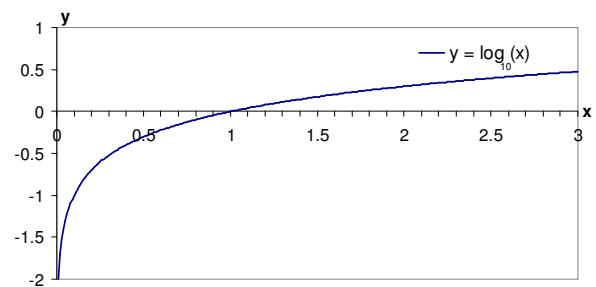
For an example see

<https://www.britannica.com/science/Richter-scale>

## § 1. Logarithms (basics)

- A **log is the power** you have to raise the base to in order to get the number
- Powers of 10 are 'logarithms' to base 10
- $\log_{10} 1000 = 3$   
 base number power ('exponent')
- $\text{antilog}_{10} 3 = 10^3 = 1000$   
 ('antilogarithm' or 'inverse logarithm')

## § 1. Logarithms (plot)



## § 1. Logarithms (rules)

$\log 1 = 0$ for any base value	$\log_{10} 1 = 0$
$\log_a a = 1$ for any value of $a$	$\log_{10} 10 = 1$
If $x = a^y$ then $\log_a x = y$	$100 = 10^2$ so $\log_{10} 100 = 2$
$\log(m \times n) = \log m + \log n$	$\log_{10}(3 \times 2) = \log_{10} 3 + \log_{10} 2$
$\log\left(\frac{m}{n}\right) = \log m - \log n$	$\log_{10}\left(\frac{3}{2}\right) = \log_{10} 3 - \log_{10} 2$
$\log m^n = n \times \log m$	$\log_{10}(3^2) = 2 \times \log_{10} 3$

## § 1. Links between logarithms and Indices (see session 3)

$\log 1 = 0$ for any base value	$a^0 = 1$ (assuming $a \neq 0$ )
$\log_a a = 1$ for any value of $a$	since $a^1 = a$
If $x = a^y$ then $\log_a x = y$	
$\log(m \times n) = \log m + \log n$	$a^m \times a^n = a^{m+n}$
$\log\left(\frac{m}{n}\right) = \log m - \log n$	$a^m \div a^n = a^{m-n}$
$\log m^n = n \times \log m$	$(a^m)^n = a^{m \times n}$

## § 1. Logarithms to different bases

- **Logarithms are simply powers** of whatever base we choose or are given, e.g.  $2^3 = 8$  so  $\log_2 8 = 3$
- Natural logarithms ( $\ln$ ) are logarithms to base  $e$  where  $e$  is a mathematical constant ( $e = 2.71828\dots$ )
- Occurrences of  $e$ :
  - Economics concept of elasticity
  - Exponential growth – e.g. for bacteria, some epidemics, population growth examples, compound interest etc
  - Exponential decay – e.g. heat loss, radioactive decay, charge on capacitor in an electronic heart pacemaker

## § 2. Transforming to a straight line (equations)

- Start with non-linear equation

$$y = 3x^2$$

Take logs

$$\log_{10} y = \log_{10}(3x^2)$$

But

$$\log_{10}(3x^2) = \log_{10} 3 + \log_{10}(x^2) \\ = \log_{10} 3 + 2\log_{10} x$$

So

$$\log_{10} y = 2\log_{10} x + \log_{10} 3$$

which is in the form  $Y = mX + c$  where

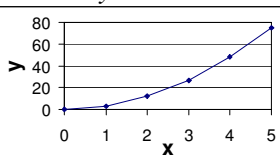
$$X = \log_{10} x, Y = \log_{10} y, m = 2 \text{ and } c = \log_{10} 3$$

- End with linear equation

## § 2. Transforming to a straight line (graphs)

Parabola (curved graph) from non-linear equation

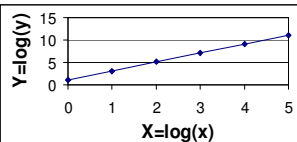
$$y = 3x^2$$



Straight line graph from linear equation

$$Y = 2X + \log_{10} 3$$

$$\log_{10} y = 2\log_{10} x + \log_{10} 3$$



## § 3. Applied problems

- Suppose blood serum concentration of protein  $P$  **doubles** if daily dose of drug  $A$  is increased by **1mg**
- If daily dose of  $A$  rises by **6mg**, what factor is concentration of  $P$  increased by? Write also as a log to base 2.
  - $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = 64 \rightarrow \log_2 64 = 6$   
 $1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$  ← mg increase in drug  $A$
- If want to raise level of  $P$  by factor of **256**, what increase in  $A$  do we need?
  - Easiest method for most students is to keep doubling until reach 256:  
 $2^6 = 64, 2^7 = 128, 2^8 = 256$   
 Need to increase  $A$  by **8 mg**

## § 4. Topics in Term 1 modules using basic maths skills

### Logarithms

- Transforming data using natural logs
- Transforming curved graphs into straight lines
- Geometric mean and relationship with arithmetic mean of logarithms

## Intended learning objectives (achieved?)

- You should be able to:
  - understand the concept of logarithms, inverse logs and natural logs
  - use the rules of logs
  - use the log function on the calculator
  - transform non-linear to straight line graphs using logs

...if not, then extra external support is available online, including a video:

<http://www.mathtutor.ac.uk/algebra/logarithms>

## Key messages

- **Logarithms are simply powers** of whatever base we chose or are given
- A log is the **power** you have to raise the **base** to in order to get the number
- When we multiply the numbers we **add** the logs and when we divide the numbers we **subtract** the logs (for logs to the same base)

$$\log(m \times n) = \log m + \log n$$

$$\log\left(\frac{m}{n}\right) = \log m - \log n$$

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**BASIC MATHS SUPPORT SESSIONS**

**Session 4 - Logarithms Notes**

**§ 1. Logarithms**

Any **positive** number can be written as a power of 10, e.g. 100 can be written as  $10^2$  or 2755 can be written as  $10^{3.44}$ .

The **powers of 10** are **logarithms to base 10**.

e.g.

Number	1000	100	10	1	0.1	0.01	0.001
Power of 10	$10^3$	$10^2$	$10^1$	$10^0$	$10^{-1}$	$10^{-2}$	$10^{-3}$
$\log_{10}$ of number	3	2	1	0	-1	-2	-3

e.g.

Number	2755	31.62
Power of 10	$10^{3.44}$	$10^{1.5}$
$\log_{10}$ of number	3.44	1.5

So, the logarithm of a number x with respect to base 10 is the power to which we have to raise 10 to obtain x.

i.e.  $\log_{10}x = y$  is equivalent to  $x = 10^y$  ( $\log_{10}$  is often referred to as log)

Note that according to above definition  $10^{\log(x)} = x$ .

▶▶ On the **CASIO fx-85GT PLUS** calculator, use the **log** key to find the logarithm of a number to base 10.

Note that if the log of 100 is 2 then 100 is the **antilogarithm** (inverse log) of 2, i.e. 100 is the number which corresponds to the log of 2, i.e. 100 corresponds to  $10^2$ .

e.g.  $\log_{10} 100 = 2$  and therefore  $\text{antilog}_{10} 2 = 100$  or  $10^2 = 100$   
10 is the base      number      power

**Natural logarithms**

Natural logarithms are logarithms to base *e* where *e* is a mathematical constant approximately equal to 2.718. We write **log<sub>e</sub> x** or **ln x** where **log<sub>e</sub> 2.718 = 1**.

e.g.  $\log_e 100 = 4.6$  and therefore  $\text{antilog}_e 4.6 = 100$   
e is the base      number      power

▶▶ On the **CASIO fx-85GT PLUS** calculator, use the **ln** key to find the natural logarithm of a number and use the **SHIFT** key followed by the **ln** key to find the inverse natural log (natural antilog) of a number. (**SHIFT** plus **ln** of a number is equivalent to finding *e* to the power of that number and you will see  $e^{\square}$  or  $e^x$  printed above the **ln** key).

**Note that logarithms to base 10 or to base e are only defined for positive numbers** so if you try to find the log of a negative number you will get a syntax error on the calculator.

**Rules of logs** definition:  $x = a^y$  is equivalent to  $\log_a x = y$

Note :  $\log(1) = 0$  for any value of base used.

$\log_a(a) = 1$  for all values of a.

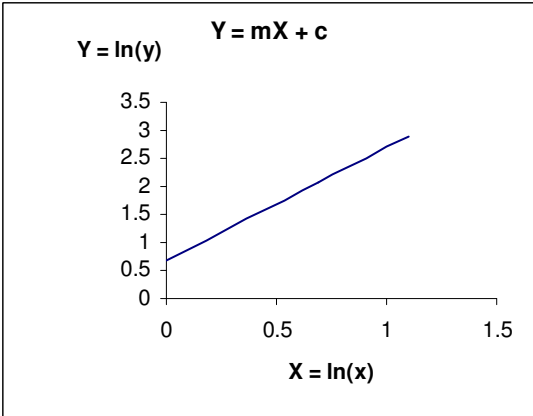
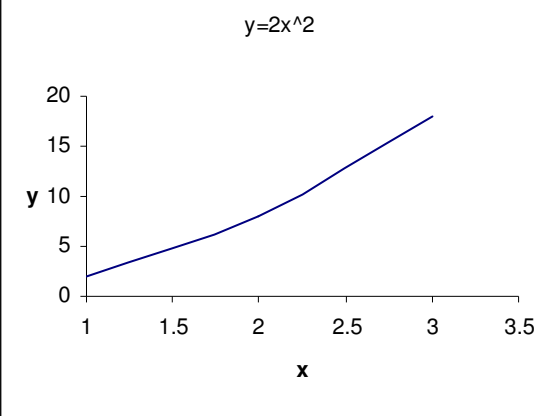
- |   |  |
|---|--|
| <p>1) <b><math>\log(m \times n) = \log m + \log n</math></b><br/>                 2) <b><math>\log(m/n) = \log m - \log n</math></b><br/>                 3) <b><math>\log m^n = n \times \log m</math></b></p> | <p>e.g. <math>\log(2 \times 3 \times 5) = \log 2 + \log 3 + \log 5</math><br/> <math>\log(2/3) = \log 2 - \log 3</math><br/> <math>\log 2^3 = 3 \times \log 2</math></p> |
|---|--|

**§ 2. Transforming to a straight line graph**

Non-linear equations can be reduced to a linear form. If we draw, for example, the graph of the equation  $y = 2x^2$  (for  $x \geq 1$ ) we obtain a parabola (curved graph). One way to transform the data is by using natural logs.

We can take logs of both sides of the equation to obtain  
 $\ln(y) = \ln(2x^2) = \ln(2) + \ln(x^2) = \ln(2) + 2\ln(x)$  (using our rules for logs)  
i.e.  $\ln(y) = 2\ln(x) + \ln(2)$

This corresponds to the form  $Y = mX + c$  where  $Y$  is equivalent to  $\ln(y)$ ,  $X$  is equivalent to  $\ln(x)$ ,  $m$  is 2 and  $c$  is  $\ln(2)$ . If we plot  $Y$  against  $X$  we will obtain a straight line. Transforming data so that we can draw a straight-line graph often enables us to deal more easily with our data set.



**London School of Hygiene & Tropical Medicine  
BASIC MATHS SUPPORT SESSIONS**

**Session 4 - Logarithms Exercises**

You do **not** need to use a calculator for these exercises.

**§ 1. Logarithms**

- 1) Write the following in logarithmic form:  
 a)  $10^4 = 10000$       b)  $10^{-4} = 0.0001$       c)  $e^2 = 7.389$
- 2) Write the following in exponential form (using powers of 10 and  $e$ )  
 a)  $\log_{10} 1000000 = 6$       b)  $\log_e 20.1 = 3$       c)  $\log_e 1 = 0$
- 3) Simplify the following in terms of logs (you do not need to work out the numerical answer)  
 a)  $\log 5 + \log 3$       b)  $\log 10 - \log 2$       c)  $\log 5^3$       d)  $\log 3^5$       e)  $\log 4 + \log 3 - \log 2$

**§ 2. Transforming to a straight line graph**

1) We can transform the equation  $y = x^3$  into a straight line form by taking logs of both sides to give  $\log(y) = 3\log(x)$

a) Complete the following table using logs to base 10:

x	1	10	100	1000
$y = x^3$				
$\log(x)$				
$\log(y)$				

- b) Plot  $\log(y)$  against  $\log(x)$  for the values in the table.  
 c) What is the gradient of the straight-line graph?

**§ 3. Applied problems**

1. Suppose that the blood serum concentration of a certain protein P doubles if the daily dose of a drug A is increased by 1mg, provided the dosage is within a certain dosage range. We will assume that in the following questions the dosage always falls within this range.

- a) By what factor is the concentration of P increased if the daily dose of A rises by 5 mg?  
 b) Express your answer to a) as a logarithm to the base 2. (Remember that  $\log_2 x$  is the power you have to raise 2 by to equal x: so  $\log_2 x = y$  such that  $2^y = x$ .)  
 c) What increase in dosage of A would be required to raise the level of P by a factor of 128?  
 d) By what factor is the level of P altered when the dose of A falls by 5 mg? Leave your answer as a fraction.  
 e) Express your answer to d) as a logarithm to base 2.  
 f) If the dose of A does not change at all, i) by what factor does P change? ii) what is this factor expressed as a logarithm to base 2?



**London School of Hygiene & Tropical Medicine  
BASIC MATHS SUPPORT SESSIONS****Session 4 - Logarithms Further Exercises**

These are intended to supplement the exercises you have already done and extend the concepts.

You **will** need a calculator for some of these exercises. If you need help with using your calculator please consult your calculator manual and see the information on calculator use offered in Basic Statistics for PHP or Statistics for EPH or Statistics with Computing.

- 1) If  $\log_{10}x^3 - \log_{10}100 = 7$  find the value of  $x$ .
- 2) If  $\log_{10}(10x^3) - \log_{10}(100/x) + \log_{10}(0.01) = 5$  find the value of  $x$ .
- 3) Find the value of the following and give your answer, where appropriate, to 2 decimal places:
  - a)  $\log_e 4.8 + \log_e 3.2 - \log_e 2.7$
  - b)  $\log_e 7^2 \div \log_e 7$
- 4) a) Using logarithms, what would you plot on each axis in order to convert the following curves into straight line graphs?
  - b) What is the gradient and intercept of the resulting straight line graphs?
    - i)  $y = 2x^5$
    - ii)  $3y = 4x^2$
    - iii)  $xy = 5$

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BASIC MATHS SUPPORT SESSIONS**

**Session 4 - Logarithms Solutions**

**§ 1. Logarithms**

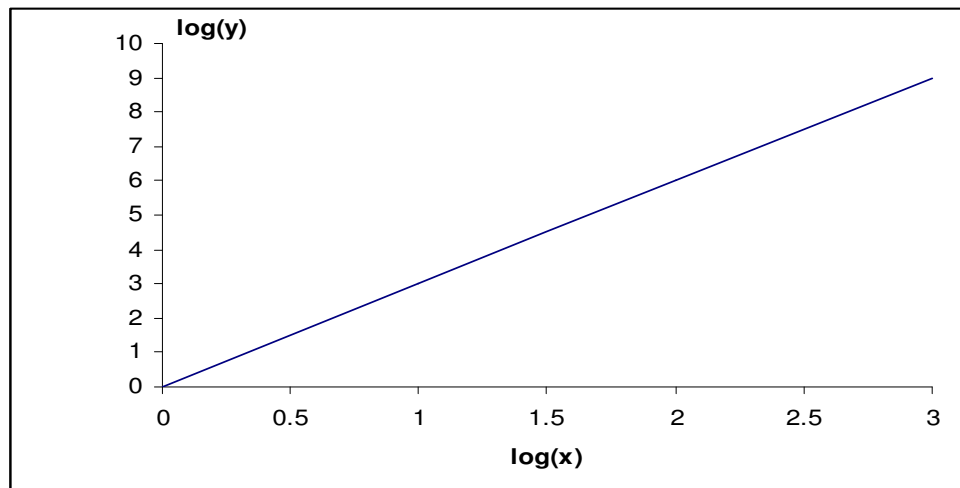
- 1) a)  $\log_{10} 10000 = 4$       b)  $\log_{10} 0.0001 = -4$       c)  $\log_e 7.389 = 2$   
 2) a)  $10^6 = 1000000$       b)  $e^3 = 20.1$       c)  $e^0 = 1$   
 3) a)  $\log(5 \times 3) = \log 15$     b)  $\log(10/2) = \log 5$     c)  $3 \times \log 5$     d)  $5 \times \log 3$     e)  $\log(4 \times 3/2) = \log 6$

**§ 2. Transforming to a straight line graph**

1) a)

x	1	10	100	1000
$y = x^3$	1	1000	1000000	1000000000
$\log(x)$	0	1	2	3
$\log(y)$	0	3	6	9

b)



c) gradient = 3

**§ 3. Applied problems**

- 1) a)  $2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$ .  
 b) Since  $32 = 2^5$ , we know directly that  $\log_2 32 = 5$ .  
 c)  $128 = 2^7$ , ie 7 ‘doublings’, which requires increasing dose by 7mg.  
 d) If dose is 1mg lower, level of P is halved. So a fall of 5 mg gives a factor of  $(1/2)^5 = 1/2^5 = 1/32$ .  
 e) We want  $\log_2$  of  $(1/2^5)$ . But since  $1/2^5 = 2^{-5}$ , by definition the answer is **-5**.  
 f) No change is a change by a factor of **1**. Since  $2^0 = 1$ , the log factor is **0**.

**London School of Hygiene & Tropical Medicine  
BASIC MATHS SUPPORT SESSIONS**

**Session 4 - Logarithms Further Exercises – Solutions**

1)  $\log_{10}x^3 - \log_{10}100 = 7$  so  $3 \times \log_{10}x - 2 = 7$ ;

$$3 \times \log_{10}x = 7 + 2 = 9; \log_{10}x = 9/3 = 3; \text{ and } x = 10^3 = 1000$$

2)  $\log_{10}(10x^3) - \log_{10}(100/x) + \log_{10}(0.01) = 5$

$$\text{so } (\log_{10}10 + \log_{10}x^3) - (\log_{10}100 - \log_{10}x) - 2 = 5;$$

$$1 + 3 \times \log_{10}x - (\log_{10}100 - \log_{10}x) - 2 = 5; 1 + 3 \times \log_{10}x - (2 - \log_{10}x) - 2 = 5;$$

$$1 + 3 \times \log_{10}x - 2 + \log_{10}x - 2 = 5; 4 \times \log_{10}x = 8; \log_{10}x = 8/4 = 2; x = 10^2 = 100$$

3) a)  $\log_e 4.8 + \log_e 3.2 - \log_e 2.7 = \log_e(4.8 \times 3.2 \div 2.7) = \log_e 5.6888888 = 1.74$

b)  $\log_e 7^2 \div \log_e 7 = \frac{2 \times \log_e 7}{\log_e 7} = 2$

4) a) For all these graphs you would plot  $\log(y)$  on one axis, typically the vertical axis, and  $\log(x)$  on the other axis, typically the horizontal axis.

- b) i) gradient is 5 and intercept is  $\log 2$   
 ii) gradient is 2 and intercept is  $\log(4/3)$   
 iii) gradient is -1 and intercept is  $\log 5$

## USE OF CALCULATORS - CASIO FX 85GT (OR SIMILAR)

Mode For simple calculations, press **[MODE] 1** to choose COMP then **[SHIFT] [MODE] 2** to choose Line10 (to make numbers display with decimals rather than as fractions).

Simple arithmetic use the **+** **-** **x** **÷** keys. Try the following,

$$2896 + 375 + 6413 = \underline{\hspace{2cm}}$$

$$23.65 + 2.10 + 18.74 + 6.43 = \underline{\hspace{2cm}}$$

$$73 - 16 + 23 + 4 - 85 = \underline{\hspace{2cm}}$$

$$17.4 \times 5.2 \times 3.1 = \underline{\hspace{2cm}}$$

$$18 \div 3 = \underline{\hspace{2cm}}$$

$$135.62 \div 10.57 = \underline{\hspace{2cm}}$$

### Use of clear keys

**[AC]** Clears calculator ready for new calculation (does not clear memories)

**[DEL]** If wrong number or operator entered during calculation, press **DEL** until wrong numbers or operators are deleted, then enter correct number and continue with calculation

### Modifying calculations you have entered

If you want to alter or insert numbers or operators in a calculation you can use the **REPLAY** arrows to move the cursor left or right within the calculation formula. The default is for what you type at the cursor to overwrite what is already there. You can change it to insert rather than overwrite by pressing **[SHIFT]** then **[DEL]** (which gives you the yellow **[INS]**)

### Example

Enter **3 + 6 x 10 =** which should give you 63 as the answer.

We will now change this to be **( 3 + 6 ) x 10** by doing the following:

Press the **<** arrow on the blue **[REPLAY]** button. This should make a cursor appear at the end of the calculation line.

Press **[SHIFT]** then **[INS]** to switch to insert rather than overwrite mode (notice how the cursor changes).

Now using the < and > arrows on the [REPLAY] button move to where you want to insert the brackets and insert them until your line reads  $(3 + 6) \times 10$  and press = to get the answer 90.

### Precedence and Brackets

$\times$  and  $\div$  are done before + and -. Otherwise calculation proceeds from left to right. Brackets are used to over-ride this rule.

$$16 \times 7 + 2 \times 5 + 14 \times 7 = \underline{\hspace{2cm}}$$

is equivalent to

$$(16 \times 7) + (2 \times 5) + (14 \times 7) = \underline{\hspace{2cm}}$$

some other exercises on precedences and brackets,

$$3 + 5 \times 6 = \underline{\hspace{2cm}} \qquad 3 + (5 \times 6) = \underline{\hspace{2cm}}$$

$$(3 + 5) \times 6 = \underline{\hspace{2cm}}$$

$$(6 + 5) \times (2 + 15 + 8) \times (6 + 4) = \underline{\hspace{2cm}}$$

$$6 + 4 \div 5 = \underline{\hspace{2cm}} \qquad 6 + (4 \div 5) = \underline{\hspace{2cm}}$$

$$(6 + 4) \div 5 = \underline{\hspace{2cm}}$$

$$24 \div 3 \times 4 = \underline{\hspace{2cm}} \qquad (24 \div 3) \times 4 = \underline{\hspace{2cm}}$$

$$24 \div (3 \times 4) = \underline{\hspace{2cm}}$$

Multiple brackets can be used

$$1317 \div ((17 + 33) \times (41 + 6)) = \underline{\hspace{2cm}}$$

### Function keys $\sqrt{\hspace{1cm}}$ $x^2$ $x^{-1}$ **log** **ln** $e^x$

Press number followed by function key (or **SHIFT** function key if function is in "yellow").

log (to base 10) of 10; enter **log** then **10** then = to get 1

square root of 4; enter  $\sqrt{\hspace{1cm}}$  then **4** then = to get 2

natural log of 5, ln of 5 enter **ln** then **5** then = to get 1.609

1/25; can do by division, or (reciprocal of 25) enter **25** then  $x^{-1}$  then = to get 0.04

exponent (natural antilog) of 2.1	enter $e^x$ (which is <b>SHIFT ln</b> ) then <b>2.1</b> = to get 8.166
square root of (4+5)	enter $4 + 5 = \sqrt{\quad} =$ gives 9, then 3 or alternatively enter $\sqrt{4 + 5} =$
natural log of 5x(8+9)	enter $5 \times (8 + 9) = \ln =$ gives 85 then 4.443 or alternatively enter $\ln 5 \times (8 + 9) =$
reciprocal of (2+6)x(3+2)	enter $(2 + 6) \times (3 + 2) = x^{-1} =$ gives 40 then 0.025 or alternatively enter $((2 + 6) \times (3 + 2))x^{-1} =$

functions can be included in a string of calculations,

$\ln 4) + \ln 9) = e^x =$  gives 3.5835, then 36 (and note that 36 is equal to 4 x 9)

Try some more; use the keys appropriately to get

square root of 25

square root of 97.49

natural log of 176, and the antilog of the result

$13^2 + 4^2 + 7^2$

$1/17 + 1/12$

(Note, for other calculators you need to experiment to find out which order you must enter the number and the operator.)

### Memory

Use of "M" memory:

Putting a number in M: **23.6 [STO] [M+]** (where [STO] equals [SHIFT] [RCL])

If memory contains a number, screen shows "M"

To clear memory M, press **0 [STO] [M+]**

To put the answer currently on screen into M, press **[STO] [M+]**

Use **[RCL] [M+]** to recall contents of memory to screen

To add the number currently on the screen to what is in the memory, press **[M+]**

Example:- **23** [STO] [M+] **27** [M+] [RCL] [M+] gives 50 (which is 23+27)

Do this example, comparing the memory results with direct addition:-

<b>617.23 x 13.67</b>	=	_____	[STO] [M+]	
<b>12.69 ÷ 13.6</b>		_____	[M+]	
<b>14 ÷ (6+7)</b>		_____	[M+]	
<b>23 - 14<sup>2</sup></b>		_____	[M+]	
Total			=	_____ [RCL] [M+]

See the manufacturers instructions on how to use “variable” memory (A, B, C, D, X, Y)

Scientific notation

$1/40000000 = 2.5^{-08}$  means  $2.5 \times 10^{-8}$

Move decimal point 8 places to left: 0.000000025

$8973 \times 25672 \times 400 =$

$9.2141942^{10}$  means  $9.2141942 \times 10^{10}$

Move decimal point 10 places to right: 92141942000

To enter a number in scientific notation:

for  $1.946 \times 10^{-3}$ , enter **1.946 [x10<sup>x</sup>] - 3**

for  $1.36 \times 10^{15}$ , enter **1.36 [x10<sup>x</sup>] 15**

Errors

eg.  $1 \div 0 =$

To clear, press [AC] and start again

## Exercises

1.  $15.7^2 + 1/8 + \sqrt{15} =$

2.  $\frac{1}{\sqrt{(4 \times 6) + 7^2}} =$

3.  $16^{2/3} + 15^{2/8} =$

4.  $\frac{25.67 - 13.12}{3.5^2 + 4.1^2} =$

5.  $\frac{(17 - 15.4)^2}{14.6} + \frac{(12 - 11.8)^2}{23.2} =$



## Solutions

<u>Simple arithmetic:</u>	9684	
	50.92	
	-1	
	280.488	
	6	
	12.83	
<u>Precedence and brackets:</u>	220	
	220	
other exercises	33	33
	48	
	2750	
	6.8	6.8
	2	
	32	32
	2	
multiple brackets	0.5604	
<u>Function Keys:</u>	5	
	9.87	
	176	
	234	
	0.142	
<u>Memory (final answer)</u>	8266.54	
<u>Exercises</u>	250.4879	
	0.01855	
	113.458	
	0.431865	
	0.17707	