

# Basic Maths

## Session 1: Basic Arithmetic

### Intended learning objectives

- At the end of this session you should be able to:
  - add, subtract, multiply and divide fractions
  - convert fractions to decimals and decimals to fractions, multiply and divide decimals by multiples of 10
  - change a fraction to a percentage, find a percentage of a number, increase/decrease a number by a percentage
  - round whole numbers and decimals
  - use ratio and dividing in proportion
  - understand the order of arithmetic operations and use some basic calculator functions

### § 1. Fractions (simplifying and expanding)

**numerator**  
**denominator**

If we multiply (or divide) BOTH numerator and denominator by the same number we don't change the value of the expression

▪ 'simplifying' or 'cancelling down'

$$\frac{6}{9} = \frac{2}{3} \quad (\div \text{ top and bottom by } 3)$$

▪ 'expanding'

$$\frac{3}{4} = \frac{6}{8} \quad (\times \text{ top and bottom by } 2)$$

### § 1. Fractions (multiplying and dividing)

Multiplying fractions

$$\frac{2}{7} \times \frac{3}{4} = \frac{2 \times 3}{7 \times 4} = \frac{6}{28} = \frac{3}{14}$$

Dividing fractions

$$\frac{1}{9} \div \frac{2}{5} = \frac{1}{9} \times \frac{5}{2} = \frac{5}{18}$$

- multiply numerators
- multiply denominators
- simplify
- invert second (to get the '**reciprocal**') and multiply by first

### § 1. Fractions (adding and subtracting)

$$\frac{2}{5} + \frac{1}{7} = \frac{2 \times 7}{5 \times 7} + \frac{1 \times 5}{7 \times 5} = \frac{14}{35} + \frac{5}{35} = \frac{14+5}{35} = \frac{19}{35}$$

- Find '**common denominator**'
- Expand both fractions
- Add or subtract numerators

### § 1. Fractions (rewriting)

- You may need to rewrite any mixed fractions as improper fractions BEFORE performing these operations
- '**mixed fraction**' to '**improper fraction**'

$$4\frac{1}{2} = \frac{4}{1} + \frac{1}{2} = \frac{(4 \times 2)}{(1 \times 2)} + \frac{1}{2} = \frac{(4 \times 2) + 1}{2} = \frac{8+1}{2} = \frac{9}{2}$$

- '**improper fraction**' to '**mixed fraction**'

$$\frac{7}{3} = \frac{(2 \times 3) + 1}{3} = \frac{(2 \times 3)}{3} + \frac{1}{3} = \frac{2}{1} + \frac{1}{3} = 2\frac{1}{3}$$

## § 2. Decimals

- Converting decimals to fractions:

$$0.7 = \frac{7}{10}, 4.61 = 4\frac{61}{100}, 7.949 = 7\frac{949}{1000}$$

- Convert fractions to decimals by dividing numerator by denominator:

$$\frac{1}{8} = 1 \div 8 = 0.125$$

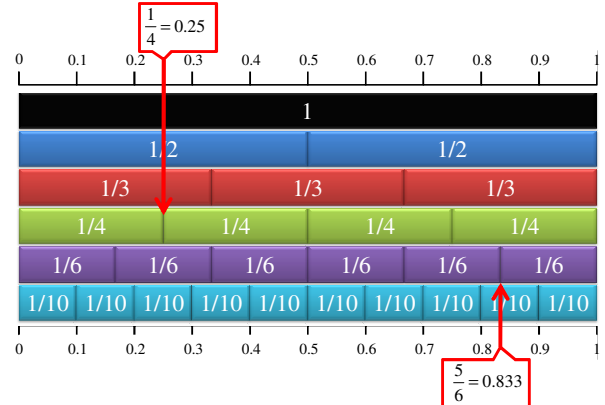
- Multiplying a decimal by a multiple of 10:

$$3.27 \times 10 = 32.7, 3.27 \times 100 = 327, 3.27 \times 1000 = 3270$$

- Dividing a decimal by a multiple of 10:

$$43.1 \div 10 = 4.31, 43.1 \div 100 = 0.431, 43.1 \div 1000 = 0.0431$$

## § 2. Fractions/Decimals in Pictures



## § 3. Percentages

- Percentages are fractions out of 100 and can be written as decimals

$$9\% = \frac{9}{100} = 0.09, 161\% = \frac{161}{100} = 1.61$$

- Converting a fraction to a percentage:

$$\frac{3}{4} \rightarrow \frac{3}{4} \times 100 = \frac{3 \times 100}{4} = \frac{3 \times 25}{1} = 75\%$$

- Percentage of a number:

$$30\% \text{ of } 60 = \frac{30}{100} \times 60 = \frac{3}{10} \times 60 = \frac{3 \times 60}{10} = \frac{3 \times 6}{1} = 18$$

- Increase/decrease a number by a percentage:

e.g. To increase 5 by 60%

$$60\% \text{ of } 5 = \frac{60}{100} \times 5 = \frac{6}{10} \times 5 = \frac{6 \times 5}{10} = \frac{6}{2} = 3, 5 + 3 = 8$$

or

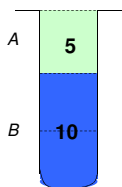
$$\frac{3 \times 5}{1} = 3, 5 + 3 = 8$$

## § 4. Rounding

Thousands Hundreds Tens Units  
7 2 8 4

- 7284 rounded to the nearest ten is 7280
- 7284 rounded to the nearest hundred is 7300
- Rounding decimals is similar e.g. 3.854 rounded to 1 decimal place is 3.9

## § 5. Ratios and proportion



- A and B are in the ratio 1:2

$$1 + 2 = 3$$

$$\frac{1}{3} \text{ of } A \text{ and } \frac{2}{3} \text{ of } B$$

- 15ml in ratio 1:2

$$\frac{1}{3} \times 15\text{ml} = \frac{15}{3}\text{ml} = 5\text{ml of } A$$

$$\frac{2}{3} \times 15\text{ml} = \frac{2 \times 15}{3}\text{ml}$$

$$= \frac{2 \times 5}{1}\text{ml} = 10\text{ml of } B$$

## § 6. Order of operations

- Do calculation from **left to right** obeying ordering:
  - Brackets (innermost 1<sup>st</sup>)
  - Exponents
  - Multiplication and Division
  - Addition and Subtraction

$$\begin{aligned} \text{E.g. } 5 + 40 \div (5 \times (12 \div 3)) &= 5 + 40 \div (5 \times 4) \\ &= 5 + 40 \div 20 \\ &= 5 + 2 \\ &= 7 \end{aligned}$$

## § 7. Applied problems

- 30ml of drug solution consists of **two thirds** drug A (costing 10p per ml), **a sixth** of drug B (costing 50p per ml) and rest of volume made up with water (no cost)
- How much does the whole solution cost?
  - A  $\rightarrow \frac{2}{3} \times 30ml = 20ml \rightarrow 20ml \times £0.10/ml = £2.00$
  - B  $\rightarrow \frac{1}{6} \times 30ml = 5ml \rightarrow 5ml \times £0.50/ml = £2.50$
  - Total cost = £2.00 + £2.50 = £4.50

- How much water is required?

$$30ml - 20ml - 5ml = 5ml \quad \text{or} \quad 1 - \frac{2}{3} - \frac{1}{6} = \frac{1}{6} \rightarrow \frac{1}{6} \times 30ml = \frac{1 \times 30}{6} ml = 5ml$$

## § 8. Topics in Term 1 modules using basic maths skills

### STATISTICS:

- Frequency
- Relative (percentage) frequency
- Arithmetic mean
- Standard deviation
- Confidence intervals
- p-values
- ...etc

### EPIDEMIOLOGY:

- Incidence
- Point prevalence (proportion)
- Risk ratio
- Rate ratio
- Odds ratio
- Mortality rates
- ...etc

## Intended learning objectives (achieved?)

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## Key messages

- If we multiply or divide BOTH numerator and denominator by the same number then the value of the fraction stays the same
- To **multiply** fractions together we multiply the top numbers together and **multiply** the bottom numbers together
- To **divide** one fraction by another we invert the one we are dividing by and then **multiply** by it
- To **add and subtract** fractions we first need to rewrite the fractions so that they have the same **denominator**
- **Percentages** are fractions out of **100**

N.B. For next session: <http://www.lshtm.ac.uk/edu/studyskills.html>  
(subheading 'Maths and Numeracy Skills')