Basic Maths

Session 1: Basic Arithmetic

Intended learning objectives
- At the end of this session you should be able to:
  - add, subtract, multiply and divide fractions
  - convert fractions to decimals and decimals to fractions, multiply and divide decimals by multiples of 10
  - change a fraction to a percentage, find a percentage of a number, increase/decrease a number by a percentage
  - round whole numbers and decimals
  - use ratio and dividing in proportion
  - understand the order of arithmetic operations and use some basic calculator functions

§ 1. Fractions (simplifying and expanding)

\[
\frac{\text{numerator}}{\text{denominator}}
\]

- ‘simplifying’ or ‘cancelling down’
- ‘expanding’

\[
\frac{6}{9} = \frac{2}{3} \quad (\times \text{top and bottom by } 3)
\]
\[
\frac{3}{4} = \frac{6}{8} \quad (\times \text{top and bottom by } 2)
\]

§ 1. Fractions (multiplying and dividing)

- multiplying fractions
  - multiply numerators
  - multiply denominators
  - simplify

\[
\frac{2}{7} \times \frac{3}{4} = \frac{6}{28} = \frac{3}{14}
\]

- dividing fractions
  - invert second (to get the ‘reciprocal’) and multiply by first

\[
\frac{1}{9} \div \frac{2}{5} = \frac{1}{9} \times \frac{5}{2} = \frac{5}{18}
\]

§ 1. Fractions (adding and subtracting)

- Find ‘common denominator’
- Expand both fractions
- Add or subtract numerators

\[
\frac{2}{5} + \frac{1}{7} = \frac{2 \times 7}{5 \times 7} + \frac{1 \times 5}{7 \times 5} = \frac{14}{35} + \frac{5}{35} = \frac{19}{35}
\]

§ 1. Fractions (rewriting)

- You may need to rewrite any mixed fractions as improper fractions BEFORE performing these operations

  - ‘mixed fraction’ to ‘improper fraction’

\[
4\frac{1}{2} = \frac{4 \times 2}{1} + \frac{1}{2} = \frac{(4 \times 2) + 1}{2} = \frac{8 + 1}{2} = \frac{9}{2}
\]

  - ‘improper fraction’ to ‘mixed fraction’

\[
\frac{7}{3} = \frac{(2 \times 3) + 1}{3} = \frac{(2 \times 3)}{3} + \frac{1}{3} = \frac{2 \times 3}{3} + \frac{1}{3} = \frac{2}{1} \frac{1}{3} = 2\frac{1}{3}
\]
§ 2. Decimals
- Converting decimals to fractions:
  \[ 0.7 = \frac{7}{10}, \quad 4.61 = 4\frac{61}{100}, \quad 7.949 = 7\frac{949}{1000} \]
- Convert fractions to decimals by dividing numerator by denominator:
  \[ \frac{1}{8} = 0.125 \]
- Multiplying a decimal by a multiple of 10:
  \[ 3.27 \times 10 = 32.7, \quad 3.27 \times 100 = 327, \quad 3.27 \times 1000 = 3270 \]
- Dividing a decimal by a multiple of 10:
  \[ 43.1 \div 10 = 4.31, \quad 43.1 \div 100 = 0.431, \quad 43.1 \div 1000 = 0.0431 \]

§ 3. Percentages
- Percentages are fractions out of 100 and can be written as decimals
  \[ 9\% = \frac{9}{100} = 0.09, \quad 161\% = \frac{161}{100} = 1.61 \]
- Converting a fraction to a percentage:
  \[ \frac{3}{4} = 3 \times \frac{100}{4} = 75\% \]
- Percentage of a number:
  \[ 30\% \text{ of } 60 = \frac{30}{100} \times 60 = \frac{3 \times 60}{10} = 18 \]
- Increase/decrease a number by a percentage:
  \[ \text{e.g. To increase } 5 \text{ by } 60\% \]
  \[ \frac{60}{100} \times 5 = \frac{6 \times 5}{10} = 3 \frac{3}{5} = 3 \frac{3}{5} \times 3 = 5 + 3 = 8 \]

§ 4. Rounding

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

- 7284 rounded to the nearest ten is 7280
- 7284 rounded to the nearest hundred is 7300

- Rounding decimals is similar e.g. 3.854 rounded to 1 decimal place is 3.9

§ 5. Ratios and proportion
- A and B are in the ratio 1:2
  \[ 1 + 2 = 3 \]
  \[ \frac{1}{3} \text{ of } A \text{ and } \frac{2}{3} \text{ of } B \]
- 15ml in ratio 1:2
  \[ \frac{1}{3} \times 15\text{ml} = 5\text{ml} = 5\text{ml of } A \]
  \[ \frac{2}{3} \times 15\text{ml} = 10\text{ml} = 10\text{ml of } B \]

§ 6. Order of operations
- Do calculation from left to right obeying ordering:
  - Brackets (innermost 1<sup>st</sup>)
  - Exponents
  - Multiplication and Division
  - Addition and Subtraction

**E.g.** 5 + 40 ÷ (5 × (12 ÷ 3)) = 5 + 40 ÷ (5 × 4) = 5 + 40 ÷ 20 = 5 + 2 = 7
§ 7. Applied problems

- 30ml of drug solution consists of two thirds drug A (costing 10p per ml), a sixth of drug B (costing 50p per ml) and rest of volume made up with water (no cost).
- How much does the whole solution cost?
  - A \rightarrow \frac{2}{3} \times 30ml = 20ml \rightarrow 20ml \times 0.10/\text{ml} = £2.00
  - B \rightarrow \frac{1}{6} \times 30ml = 5ml \rightarrow 5ml \times 0.50/\text{ml} = £2.50
  - Total cost = £2.00 + £2.50 = £4.50
- How much water is required?
  \[ 30ml - 20ml - 5ml = 5ml \]
  \[ \frac{\frac{2}{3}}{3} \times 6 \rightarrow \frac{1}{6} \times 30ml = \frac{1 \times 30}{6} = 5ml \]

§ 8. Topics in Term 1 modules using basic maths skills

STATISTICS:
- Frequency
- Relative (percentage) frequency
- Arithmetic mean
- Standard deviation
- Confidence intervals
- p-values
- …etc

EPIDEMIOLOGY:
- Incidence
- Point prevalence (proportion)
- Risk ratio
- Rate ratio
- Odds ratio
- Mortality rates
- …etc

Intended learning objectives (achieved?)

- You should be able to:
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Key messages

- If we multiply or divide BOTH numerator and denominator by the same number then the value of the fraction stays the same
- To multiply fractions together we multiply the top numbers together and **multiply** the bottom numbers together
- To divide one fraction by another we invert the one we are dividing by and then **multiply** by it
- To add and subtract fractions we first need to rewrite the fractions so that they have the same denominator
- Percentages are fractions out of 100

N.B. For next session: [http://www.lshtm.ac.uk/edu/studyskills.html](http://www.lshtm.ac.uk/edu/studyskills.html) (subheading ‘Maths and Numeracy Skills’)