

Basic Maths

Session 2: Basic Algebra

Intended learning objectives

- At the end of this session you should be able to:
 - substitute numbers for letters in algebraic expressions
 - multiply out brackets and use factorisation
 - solve simple equations
 - use and rearrange simple formulae
 - solve simple inequalities

§ 1. Algebraic expressions (indices and roots)

$$3 \times 3 \times 3 \times 3 = 3^4$$

← 'index' 'power' 'exponent'
← 'base'

$n \times n = n^2$ 'n squared' or 'n to the power 2'

$n \times n \times n = n^3$ 'n cubed' or 'n to the power 3'

$n \times n \times n \times n = n^4$ 'n to the power 4'

Roots can be used to undo indices:

Square root: $\sqrt[n^2]{n^2} = n$, (usually written as $\sqrt{n^2} = n$)

Cube root: $\sqrt[3]{n^3} = n$

Fourth root: $\sqrt[4]{n^4} = n$, and so on

§ 1. Algebraic expressions (substitution, + - × ÷ terms)

- 'Substitution': If $x = 3$ and $y = 6$

$$5x - 2y = (5 \times 3) - (2 \times 6) = 15 - 12 = 3$$

- Adding and subtracting like terms:

$$6a + 4b - a + 7b = (6 - 1)a + (4 + 7)b = 5a + 11b$$

- Multiplying and dividing algebraic terms:

$$\frac{2p^3 \times 5q}{4p} = \frac{2 \times p \times p \times p \times 5 \times q}{4 \times p} = \frac{(2 \times 5) \times p \times p \times q}{4} = \frac{10p^2q}{4} = \frac{5p^2q}{2}$$

- Algebraic fractions:

$$\frac{3}{x} - \frac{9}{4y} = \frac{(3 \times 4y) - (9 \times x)}{4xy} = \frac{12y - 9x}{4xy}$$

§ 1. Algebraic expressions (multiplying out, factorisation)

- Multiplying out brackets:

$$3(x + 2y) = 3 \times (x + 2y) = 3 \times x + 3 \times 2 \times y = 3x + 6y$$

$$-2(5x + y) = -2 \times (5x + y) = (-2) \times 5 \times x + (-2) \times y = -10x - 2y$$

$$(2x + y)(3x + 4y) = 2x \times 3x + 2x \times 4y + y \times 3x + y \times 4y$$

$$= 6x^2 + 8xy + 3xy + 4y^2 = 6x^2 + 11xy + 4y^2$$

- Factorisation:

$$3x - 6y = 3(x - 2y)$$

$$3x + xy - 2xz = x(3 + y - 2z)$$

§ 2. Simple equations (solving)

$$-\frac{2}{3}x + \frac{4}{5} = \frac{1}{3}x - 2$$

Find x :

$$\frac{4}{5} = \frac{1}{3}x + \frac{2}{3}x - 2$$

$$2 + \frac{4}{5} = \frac{1}{3}x + \frac{2}{3}x$$

$$\frac{10 + 4}{5} = \left(\frac{1 + 2}{3}\right)x$$

$$\frac{14}{5} = x$$

§ 3. Formulae (basics)

- A formula is an equation that describes the relationship between two or more quantities
- Suppose $Q = 1.4P + 3$
- If $P = 2$
 $Q = 1.4 \times 2 + 3 = 2.8 + 3 = 5.8$

§ 3. Formulae (rearranging)

- Rearrange this formula to make P the subject:

$$T = \sqrt[3]{\frac{4P}{P-Q}}$$

$$T^3 = \frac{4P}{P-Q}$$

$$T^3(P-Q) = 4P$$

$$T^3P - T^3Q = 4P$$

$$T^3P - 4P = T^3Q$$

$$P(T^3 - 4) = T^3Q$$

$$P = \frac{T^3Q}{T^3 - 4}$$

§ 4. Simple inequalities ($><\geq\leq$)

- Greater than: $>$
- Less than: $<$
- Greater than or equal to: \geq
- Less than or equal to: \leq

§ 4. Simple inequalities (solving)

$$3x - 5 \geq 7x + 8$$

$$3x - 7x \geq 8 + 5$$

$$-4x \geq 13$$

$$x \leq \frac{13}{-4}$$

(note inequality sign change when \div by negative number)

$$x \leq -\frac{13}{4}$$

§ 5. Applied problems

- Suppose there are N people of which I are infected with some disease and the rest are susceptible (S)
- Write the formula connecting N , I and S , with N as the subject
 $> N = I + S$
- What proportion (p) of people are infected?
 $> p = \frac{I}{N}$
- Write p in terms of S and N
 $> p = \frac{N-S}{N}$

§ 5. Applied problems (cont.)

- Make N the subject of this formula for p

$$p = \frac{N-S}{N}$$

$$Np = N - S$$

$$Np - N = -S$$

$$N(p-1) = -S$$

$$N = \frac{-S}{(p-1)}$$

$$N = \frac{S}{(1-p)}$$

or

$$Np + S = N$$

$$S = N - Np$$

$$S = N(1-p)$$

$$\frac{S}{(1-p)} = N$$

Note that p is a proportion so $0 \leq p \leq 1$ and $(p-1) \leq 0$

§ 6. Topics in Term 1 modules using basic maths skills

Formulae

- Calculating test statistics
(e.g. z-test using formula for standard error)
- Calculating confidence intervals
- Calculating correlation coefficient
- Standardised mortality ratios

Inequalities

- Categorising variables
- Determining significance using p-values

Intended learning objectives (achieved?)

- You should be able to:
 - substitute numbers for letters in algebraic expressions
 - multiply out brackets and use factorisation
 - solve simple equations
 - use and rearrange simple formulae
 - solve simple inequalities

Key messages

- Algebra is about making **letters** represent quantities
- We can add and **subtract** like terms
- We can multiply and **divide** algebraic terms
- **Factorisation** is the reverse of multiplying out brackets
- To solve a simple equation or **inequality** we need to find the value of the unknown quantity which is represented by the letter
- To rearrange a formula:
Remove roots; clear fractions and **brackets**; collect terms involving the required subject; factorise if necessary; isolate the required subject

N.B. For next session: <http://www.lshtm.ac.uk/edu/studyskills.html>
(subheading 'Maths and Numeracy Skills')