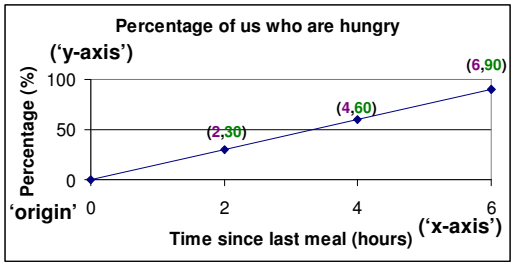
Basic Maths

Session 3: Graphs, Problem Solving, and Powers

Intended learning objectives

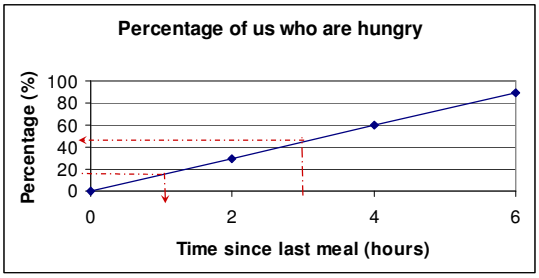
- At the end of this session you should be able to:
  - understand the terminology of graphs and use axes, scales and co-ordinates
  - plot simple graphs
  - understand the equation of a straight line and use it to plot straight line graphs
  - understand and solve problems involving unit quantities
  - understand and solve problems using probability trees
  - use the rules for indices (multiply and divide powers, raise a power to a power, reciprocals)
  - understand what is meant by standard form and convert numbers to standard form

§ 1. Plotting graphs (basics)



Time since last meal (hours)	0	2	4	6
Percentage of us hungry (%)	0	30	60	90

§ 1. Plotting graphs (interpolation)



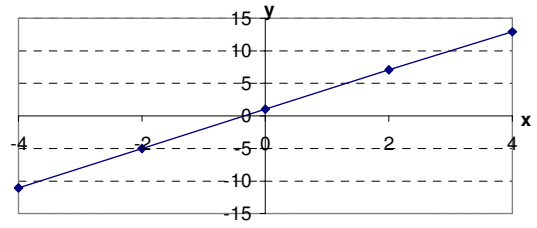
§ 2. Equation of a straight line

$y = mx + c$

'gradient' 'intercept'

$y = 3x + 1$

x	-4	-2	0	2	4
3x	-12	-6	0	6	12
+1	+1	+1	+1	+1	+1
y	-11	-5	1	7	13



§ 3. Problem solving (units – easy!)

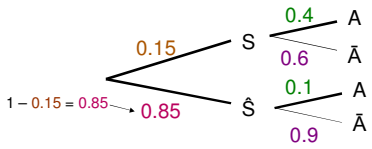
- 4 drinks cost £12
- How much do 5 drinks cost?
  - Unit is a drink
  - 1 drink costs ***less than*** 4 drinks so ***divide*** cost by 4
  - 1 drink costs $\frac{£12}{4} = £3$
  - 5 drinks cost ***more than*** 1 drink so ***multiply*** cost by 5
  - 5 drinks cost $£3 \times 5 = £15$

§ 3. Problem solving (units – hard!)

- It takes 24 weeks for 9 people to build 3 primary health centres (PHCs)
- How long does it take 4 people to build 6 PHCs?
  - First make PHC the unit and calculate how many weeks it takes 9 people to build 1 PHC
    - $\frac{24}{3} = 8$ weeks
  - Next make the number of people the unit and calculate how many weeks it takes 1 person to build 1 PHC
    - $8 \times 9 = 72$ weeks
  - Finally get answer by multiplying by the number of PHCs (6) and dividing by the number of people (4)
    - $\frac{72 \times 6}{4} = 108$ weeks for 4 people to build 6 PHCs

§ 3. Problem solving (probabilities)

- Suppose 15% of people are smokers and 40% of smokers get condition A while only 10% of non-smokers get condition A
- Out of 1000 people, how many would we expect to get condition A?



S – smoker
Ŝ – non-smoker
A – got condition A
Ā – not got condition A

$1000 \times (0.15 \times 0.4 + 0.85 \times 0.1) = 1000 \times 0.145 = 145$

§ 4. Algebraic expressions (indices and roots)

$3 \times 3 \times 3 \times 3 = 3^4$ ← 'index' 'power' 'exponent'
1 2 3 4 ← 'base'

$n \times n = n^2$ 'n squared' or 'n to the power 2'

$n \times n \times n = n^3$ 'n cubed' or 'n to the power 3'

$n \times n \times n \times n = n^4$ 'n to the power 4'

Roots can be used to undo indices:

Square root : $\sqrt[2]{n^2} = n$, (usually written as $\sqrt{n^2} = n$)

Cube root : $\sqrt[3]{n^3} = n$

Fourth root : $\sqrt[4]{n^4} = n$, and so on

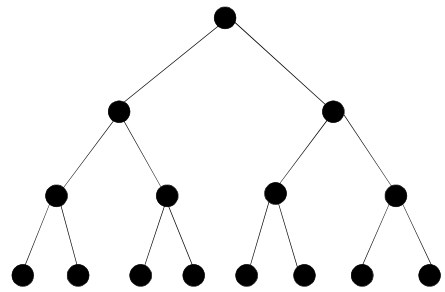
§ 4. Indices (doubling)

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$



§ 4. Indices (rules)

$a^m \times a^n = a^{m+n}$	$4^3 \times 4^2 = (4 \times 4 \times 4) \times (4 \times 4) = 4^5 = 4^{3+2}$
$a^m \div a^n = a^{m-n}$	$\frac{4^3}{4^2} = \frac{4 \times 4 \times 4}{4 \times 4} = 4 = 4^1 = 4^{3-2}$
$(a^m)^n = a^{m \times n}$	$(4^3)^2 = (4 \times 4 \times 4)^2 = (4 \times 4 \times 4) \times (4 \times 4 \times 4) = 4^6 = 4^{3 \times 2}$
$a^{-m} = \frac{1}{a^m}$	$4^{-3} = \frac{1}{4^3}$

§ 4. Indices (more rules!)

$a^0 = 1$ (assuming $a \neq 0$)	$4^0 = 1$
$a^{1/n} = \sqrt[n]{a}$	$4^{1/2} = \sqrt{4}$
$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$	$4^{3/2} = \sqrt{4^3} = \sqrt{64} = 8$ and $4^{3/2} = \left(\sqrt{4}\right)^3 = (2)^3 = 8$
$(a \times b)^n = a^n \times b^n$	$(4 \times 8)^2 = (4 \times 8) \times (4 \times 8) = 4 \times 4 \times 8 \times 8 = 4^2 \times 8^2$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{4}{8}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ and $\frac{4^2}{8^2} = \frac{16}{64} = \frac{1}{4}$

§ 4. Roots (just two more!)

(assuming $a \geq 0$ and $b \geq 0$)

$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$$

$$\sqrt[3]{27 \times 64} = \sqrt[3]{1728} = 12 \text{ and } \sqrt[3]{27} \times \sqrt[3]{64} = 3 \times 4 = 12$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \text{ (assuming } b \neq 0)$$

$$\sqrt{\frac{81}{9}} = \sqrt{9} = 3 \text{ and } \frac{\sqrt{81}}{\sqrt{9}} = \frac{9}{3} = 3$$

These rules are used in the Basic Statistics module

§ 4. Indices (standard form)

$$4{,}000{,}000{,}000 = 4 \times 10^{9}$$

$$23{,}950 = 2.395 \times 10^{4}$$

$$0.00648 = 6.48 \times 10^{-3}$$

$$4 \times 10^{5} - 5 \times 10^{3} = (4 \times 100{,}000) - (5 \times 1{,}000)$$
$$= 400{,}000 - 5{,}000 = 395{,}000$$

$$\frac{4 \times 10^{7}}{2 \times 10^{3}} = \frac{4}{2} \times 10^{7-3} = 2 \times 10^{4}$$

§ 5. Topics in Term 1 modules using basic maths skills

Graphs
- Descriptive statistics (visual representation of relationship between variables)
- Linear regression

Problem solving
- Applying basic maths skills
- Thinking through appropriate strategies using these skills

Powers and square root
- Variance
- Standard deviation
- Standard error

Standard form
- Calculator readout

Intended learning objectives (achieved?)

- You should be able to:
  - understand the terminology of graphs and use axes, scales and co-ordinates
  - plot simple graphs
  - understand the equation of a straight line and use it to plot straight line graphs
  - understand and solve problems involving unit quantities
  - understand and solve problems using probability trees
  - use the rules for indices (multiply and divide powers, raise a power to a power, reciprocals)
  - understand what is meant by standard form and convert numbers to standard form

Key rules of powers

- To multiply (quantities to) powers **OF THE SAME BASE** **add** the indices
- To divide (quantities to) powers **OF THE SAME BASE** **subtract** the indices
- To raise a power of a quantity to a power, **multiply** the indices
- A negative index gives the **reciprocal** of the quantity

N.B. For next session: http://www.lshtm.ac.uk/edu/studyskills.html (subheading 'Maths and Numeracy Skills')