Basic Maths

Session 4: Logarithms

Intended learning objectives

- At the end of this session you should be able to:
  - ➢understand the concept of logarithms, inverse logs and natural logs
  - ➢use the rules of logs
  - ➢use the log function on the calculator
  - ➢transform non-linear to straight line graphs using logs

§ 1. Logarithms (activity – part 1)

- Plot the following coordinate points on graph paper

x	100	1,000	10,000	100,000	1,000,000
y	0.001	0.0001	0.00001	0.000001	0.0000001

§ 1. Logarithms (basics)

- A **log is the power** you have to raise the base to in order to get the number
- **Powers** of 10 are '**logarithms**' to base 10
- $\log_{10} 1000 = 3$

  base number power ('exponent')
- $\text{antilog}_{10} 3 = 10^3 = 1000$

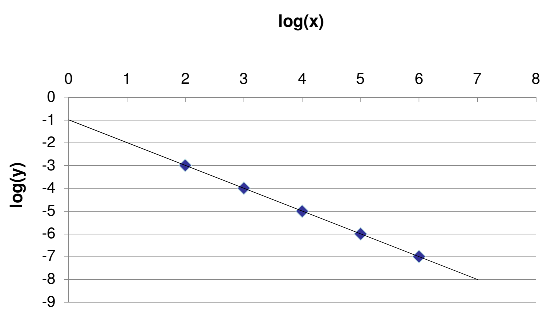
  ('**antilogarithm**' or '**inverse logarithm**')

§ 1. Logarithms (activity – part 2)

- Use log button on calculator to convert (*x,y*) coordinates and plot (*log(x),log(y)*) on graph paper

x	100	1,000	10,000	100,000	1,000,000
log(x)	2	3	4	5	6
y	0.001	0.0001	0.00001	0.000001	0.0000001
log(y)	-3	-4	-5	-6	-7

§ 1. Logarithms (activity – part 2)



§ 1. Logarithms (uses)

- Logarithms make very large or very small numbers easier to handle
- Logarithms convert quite complicated mathematical manipulations into easier forms
- Logarithms can be used to convert curved graphs into straight-line graphs to determine the exact values in the relationship between variables

§ 1. Logarithms and exponentials (applications)

Examples of uses of logarithms

- Richter scale for earthquakes uses logarithm scale
- pH scale for acidity of substances
- Exponential and logistic population growth models
- Exponential decay of drug concentration in a patient's body
- Decibel scale for the power of sound uses logarithm scale

For an example see

https://www.britannica.com/science/Richter-scale

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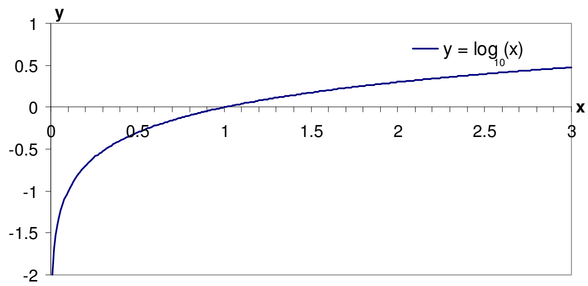
§ 1. Logarithms (plot)



§ 1. Logarithms (rules)

$\log 1 = 0$ for any base value	$\log_{10} 1 = 0$
$\log_a a = 1$ for any value of a	$\log_{10} 10 = 1$
If $x = a^y$ then $\log_a x = y$	$100 = 10^2$ so $\log_{10} 100 = 2$
$\log(m \times n) = \log m + \log n$	$\log_{10}(3 \times 2) = \log_{10} 3 + \log_{10} 2$
$\log\left(\frac{m}{n}\right) = \log m - \log n$	$\log_{10}\left(\frac{3}{2}\right) = \log_{10} 3 - \log_{10} 2$
$\log m^n = n \times \log m$	$\log_{10}(3^2) = 2 \times \log_{10} 3$

§ 1. Links between logarithms and Indices (see session 3)

$\log 1 = 0$ for any base value	$a^0 = 1$ (assuming $a \neq 0$)
$\log_a a = 1$ for any value of a	since $a^1 = a$
If $x = a^y$ then $\log_a x = y$	
$\log(m \times n) = \log m + \log n$	$a^m \times a^n = a^{m+n}$
$\log\left(\frac{m}{n}\right) = \log m - \log n$	$a^m \div a^n = a^{m-n}$
$\log m^n = n \times \log m$	$(a^m)^n = a^{m \times n}$

§ 1. Logarithms to different bases

- Logarithms are simply powers of whatever base we choose or are given, e.g. $2^3 = 8$ so $\log_2 8 = 3$
- Natural logarithms (ln) are logarithms to base *e* where *e* is a mathematical constant (*e* = 2.71828…)
- Occurrences of *e*:
  - Economics concept of elasticity
  - Exponential growth – e.g. for bacteria, some epidemics, population growth examples, compound interest etc
  - Exponential decay – e.g. heat loss, radioactive decay, charge on capacitor in an electronic heart pacemaker

§ 2. Transforming to a straight line (equations)

- Start with non-linear equation

$$y = 3x^2$$

Take logs $\log_{10} y = \log_{10}(3x^2)$

But $\log_{10}(3x^2) = \log_{10} 3 + \log_{10}(x^2)$

$$= \log_{10} 3 + 2\log_{10} x$$

So $\log_{10} y = 2\log_{10} x + \log_{10} 3$

which is in the form $Y = mX + c$ where

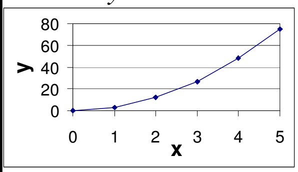
$X = \log_{10} x$, $Y = \log_{10} y$, $m = 2$ and $c = \log_{10} 3$

- End with linear equation

§ 2. Transforming to a straight line (graphs)

Parabola (curved graph) from non-linear equation

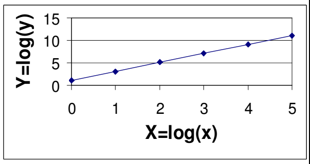
$$y = 3x^2$$



Straight line graph from linear equation

$$Y = 2X + \log_{10} 3$$

$$\log_{10} y = 2\log_{10} x + \log_{10} 3$$



§ 3. Applied problems

- Suppose blood serum concentration of protein *P* doubles if daily dose of drug *A* is increased by 1mg
- If daily dose of *A* rises by 6mg, what factor is concentration of *P* increased by? Write also as a log to base 2.
  - $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = 64 \rightarrow \log_2 64 = 6$

    1 2 3 4 5 6 ← mg increase in drug A
- If want to raise level of *P* by factor of 256, what increase in *A* do we need?
  - Easiest method for most students is to keep doubling until reach 256:

    $2^6 = 64$, $2^7 = 128$, $2^8 = 256$

    Need to increase A by 8 mg

§ 4. Topics in Term 1 modules using basic maths skills

Logarithms

- Transforming data using natural logs
- Transforming curved graphs into straight lines
- Geometric mean and relationship with arithmetic mean of logarithms

Intended learning objectives (achieved?)

- You should be able to:
  - understand the concept of logarithms, inverse logs and natural logs
  - use the rules of logs
  - use the log function on the calculator
  - transform non-linear to straight line graphs using logs

…if not, then extra external support is available online, including a video:

http://www.mathtutor.ac.uk/algebra/logarithms

Key messages

- Logarithms are simply powers of whatever base we chose or are given
- A log is the **power** you have to raise the **base** to in order to get the number
- When we multiply the numbers we **add** the logs and when we divide the numbers we **subtract** the logs (for logs to the same base)

$$\log(m \times n) = \log m + \log n$$

$$\log\left(\frac{m}{n}\right) = \log m - \log n$$