

# Basic Maths

## Session 4: Logarithms

### Intended learning objectives

- At the end of this session you should be able to:
  - understand the concept of logarithms, inverse logs and natural logs
  - use the rules of logs
  - use the log function on the calculator
  - transform non-linear to straight line graphs using logs

### § 1. Logarithms (activity – part 1)

- Plot the following coordinate points on graph paper

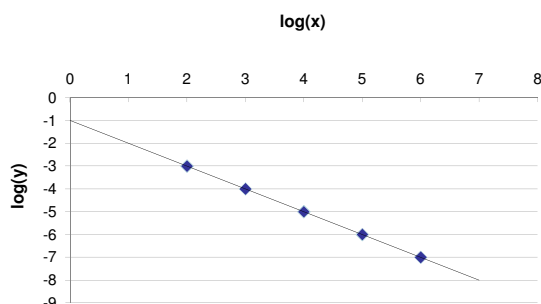
x	100	1,000	10,000	100,000	1,000,000
y	0.001	0.0001	0.00001	0.000001	0.0000001

### § 1. Logarithms (activity – part 2)

- Use log button on calculator to convert  $(x,y)$  coordinates and plot  $(\log(x), \log(y))$  on graph paper

x	100	1,000	10,000	100,000	1,000,000
$\log(x)$	2	3	4	5	6
y	0.001	0.0001	0.00001	0.000001	0.0000001
$\log(y)$	-3	-4	-5	-6	-7

### § 1. Logarithms (activity – part 2)



### § 1. Logarithms (uses)

- Logarithms make very large or very small numbers easier to handle
- Logarithms convert quite complicated mathematical manipulations into easier forms
- Logarithms can be used to convert curved graphs into straight-line graphs to determine the exact values in the relationship between variables

## § 1. Logarithms and exponentials (applications)

### Examples of uses of logarithms

- Richter scale for earthquakes uses logarithm scale
- pH scale for acidity of substances
- Exponential and logistic population growth models
- Exponential decay of drug concentration in a patient's body
- Decibel scale for the power of sound uses logarithm scale

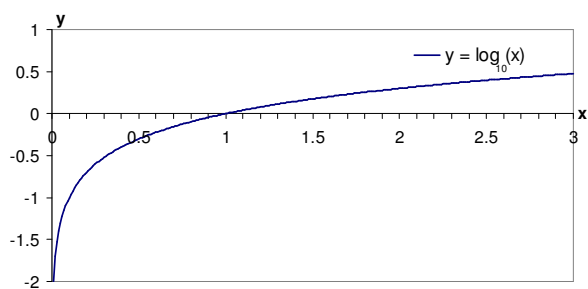
For examples see

- <http://scc.scdsb.edu.on.ca/Students/onlinecourses/Sacchetto/AFIC%20web%20page/pdf%20files/7-5%20Logarithmic%20Scales.pdf>
- <http://highered.mcgraw-hill.com/sites/dl/free/0072867388/374777/ch05.pdf>

## § 1. Logarithms (basics)

- A **log is the power** you have to raise the base to in order to get the number
- Powers of 10 are '**logarithms**' to base 10
- $\log_{10} 1000 = 3$   
 base number power ('exponent')
- $\text{antilog}_{10} 3 = 10^3 = 1000$   
 ('antilogarithm' or 'inverse logarithm')

## § 1. Logarithms (plot)



## § 1. Logarithms (rules)

$\log 1 = 0$ for any base value	$\log_{10} 1 = 0$
$\log_a a = 1$ for any value of $a$	$\log_{10} 10 = 1$
If $x = a^y$ then $\log_a x = y$	$100 = 10^2$ so $\log_{10} 100 = 2$
$\log(m \times n) = \log m + \log n$	$\log_{10}(3 \times 2) = \log_{10} 3 + \log_{10} 2$
$\log\left(\frac{m}{n}\right) = \log m - \log n$	$\log_{10}\left(\frac{3}{2}\right) = \log_{10} 3 - \log_{10} 2$
$\log m^n = n \times \log m$	$\log_{10}(3^2) = 2 \times \log_{10} 3$

## § 1. Links between logarithms and Indices (see session 3)

$\log 1 = 0$ for any base value	$a^0 = 1$ (assuming $a \neq 0$ )
$\log_a a = 1$ for any value of $a$	since $a^1 = a$
If $x = a^y$ then $\log_a x = y$	
$\log(m \times n) = \log m + \log n$	$a^m \times a^n = a^{m+n}$
$\log\left(\frac{m}{n}\right) = \log m - \log n$	$a^m \div a^n = a^{m-n}$
$\log m^n = n \times \log m$	$(a^m)^n = a^{m \times n}$

## § 1. Logarithms to different bases

- **Logarithms are simply powers** of whatever base we choose or are given, e.g.  $2^3 = 8$  so  $\log_2 8 = 3$
- Natural logarithms (ln) are logarithms to base  $e$  where  $e$  is a mathematical constant ( $e = 2.71828\dots$ )
- Occurrences of  $e$ :
  - Economics concept of elasticity
  - Exponential growth – e.g. for bacteria, some epidemics, population growth examples, compound interest etc
  - Exponential decay – e.g. heat loss, radioactive decay, charge on capacitor in an electronic heart pacemaker

More Information:

<http://highered.mcgraw-hill.com/sites/dl/free/0072867388/374777/ch05.pdf>

## § 2. Transforming to a straight line (equations)

- Start with non-linear equation

$$y = 3x^2$$

Take logs  $\log_{10} y = \log_{10}(3x^2)$

But  $\log_{10}(3x^2) = \log_{10} 3 + \log_{10}(x^2)$   
 $= \log_{10} 3 + 2\log_{10} x$

So  $\log_{10} y = 2\log_{10} x + \log_{10} 3$

which is in the form  $Y = mX + c$  where

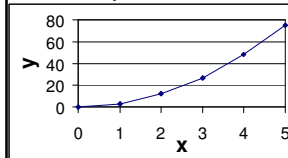
$X = \log_{10} x$ ,  $Y = \log_{10} y$ ,  $m = 2$  and  $c = \log_{10} 3$

- End with linear equation

## § 2. Transforming to a straight line (graphs)

Parabola (curved graph) from non-linear equation

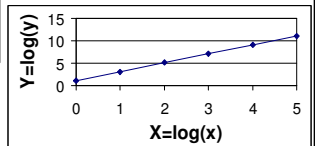
$$y = 3x^2$$



Straight line graph from linear equation

$$Y = 2X + \log_{10} 3$$

$$\log_{10} y = 2\log_{10} x + \log_{10} 3$$



## § 3. Applied problems

- Suppose blood serum concentration of protein  $P$  **doubles** if daily dose of drug  $A$  is increased by **1 mg**
- If daily dose of  $A$  rises by **6 mg**, what factor is concentration of  $P$  increased by? Write also as a log to base **2**.
  - $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = 64 \rightarrow \log_2 64 = 6$   
 1 2 3 4 5 6 ← mg increase in drug  $A$
- If want to raise level of  $P$  by factor of **256**, what increase in  $A$  do we need?
  - Easiest method for most students is to keep doubling until reach 256:  
 $2^6 = 64$ ,  $2^7 = 128$ ,  $2^8 = 256$   
 Need to increase  $A$  by **8 mg**

## § 4. Topics in Term 1 modules using basic maths skills

### Logarithms

- Transforming data using natural logs
- Transforming curved graphs into straight lines
- Geometric mean and relationship with arithmetic mean of logarithms

## Intended learning objectives (achieved?)

- You should be able to:
  - understand the concept of logarithms, inverse logs and natural logs
  - use the rules of logs
  - use the log function on the calculator
  - transform non-linear to straight line graphs using logs

...if not, then extra external support is available online, including a video:

<http://www.mathstutor.ac.uk/algebra/logarithms>

## Key messages

- Logarithms are simply powers** of whatever base we chose or are given
- A log is the **power** you have to raise the **base** to in order to get the number
- When we multiply the numbers we **add** the logs and when we divide the numbers we **subtract** the logs (for logs to the same base)

$$\log(m \times n) = \log m + \log n$$

$$\log\left(\frac{m}{n}\right) = \log m - \log n$$